FAIA 2025 is the continuation of our biannual conference *The International Mathematical Conference "Functional Analysis in Interdisciplinary Applications"*.
Previous conferences were held in Astana (Kazakhstan) in October 2–5, 2017; Lefkosa (Cyprus) in September 6–9, 2018; Antalya (Türkiye) in October 2–7, 2023.
The conference aims are to join mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. As the knowledge of different branches of mathematics open new perspectives, it is important to learn more about the developments and advancements in the field of applied mathematics and analysis. As organizers we are proud to see that FAIA provides a forum for researchers and scientists to share their recent developments and to present their original results in various fields of mathematics.
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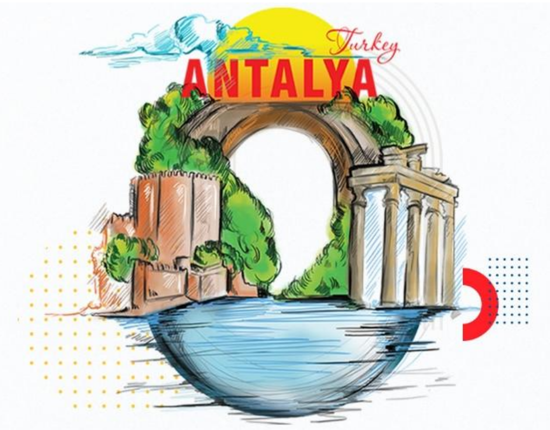

FAIA 2025

Sept. 6-13, 2025
Antalya (Manavgat), Türkiye

International Mathematical Conference "Functional Analysis in Interdisciplinary Applications"

ABSTRACT BOOK

International Mathematical Conference

Functional Analysis in Interdisciplinary Applications”

ABSTRACT BOOK

of the conference FAIA 2025

Edited by

Charyyar Ashyralyyev,

Makhmud A. Sadybekov

September 6 –13, 2025

Bahcesehir University,
Istanbul, Türkiye

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Almaty, Kazakhstan

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We, the participants of International Mathematical Conference ”Functional Analysis in Interdisciplinary Applications” (FAIA 2025), all are very blessed to meet in-person after the pandemic and this abstract book is the valuable outcome of this gathering. As organizers, we are also fortunate because we received a very high number of abstracts submitted.

FAIA 2025 is the continuation of our biannual conference *The International Mathematical Conference ”Functional Analysis in Interdisciplinary Applications”* (FAIA).

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FOREWORD

On behalf of the Organizing Committee, we are pleased to invite you to the International Mathematical Conference "Functional Analysis in Interdisciplinary Applications", FAIA 2025. The meeting will be held in September 6 - 13, 2025 in Antalya, Türkiye. The conference will consist of plenary lectures and contributed oral presentations.

Previous conferences were held in Astana, Kazakhstan in October 2–5, 2017, Lefkosa, Cyprus in September 6–9, 2018, Antalya, Türkiye in October 2–7, 2023.

Selected full papers of this conference will be published in peer-reviewed journals "Bulletin of the Karaganda University-Mathematics" (https://mathematics-vestnik.ksu.kz/) and "e-Journal of Analysis and Applied Mathematics" (https://ejaam.org/home)

We would like to thank our main sponsors Bahcesehir University, Türkiye, Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Ghent Analysis & PDE Center, Belgium. We also would like to thank all participants, Local Organizing and Technical Program Committee Members.

With our best wishes and warm regards,

Prof. Allaberen Ashyralyev

Prof. Michael Ruzhansky

Prof. Makhmud Sadybekov

Chairs of FAIA 2025

GENERAL SECTION

The Fujita exponent for a heat equation with mixed local and nonlocal nonlinearities

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Abstract: We study a semilinear heat equation involving a mixed local and nonlocal nonlinearity. First, we establish the local existence and uniqueness of mild solutions for regular, nonnegative initial data. We then prove the global existence and nonexistence of solutions under suitable growth conditions on the nonlinear terms. This leads to the identification of the Fujita exponent.

Throughout this note we mainly use techniques from these works [1, 2].

This research was funded by the Research Group Unit, College of Engineering and Technology, American University of the Middle East

Keywords: Nonlinear parabolic equations, local/global existence, finite-time blow-up, nonlinear memory, nonlinear reaction, Fujita critical exponent,

2020 Mathematics Subject Classification: 35K55, 35B44, 35A01, 26A33

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Orthoisomorphisms of idempotents of certain unital C^*-algebras

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Abstract: Al-Rawashdeh studied the induced map θ_φ between the projections and he proved that it is an orthoisomorphism for a large class of C^*-algebras. In this paper, we extend the results of Al-Rawashdeh by replacing $\mathcal{U}(\mathcal{A})$ by $GL(\mathcal{A})$, and $\mathcal{P}(\mathcal{A})$ by $\mathcal{I}(\mathcal{A})$, and we prove that θ_φ is an orthoisomorphism of idempotents for certain type of UHF algebras, C^*-algebras of 2-divisible K_0-groups, Cuntz algebras, and for simple, unital purely infinite C^*-algebras having 2-divisible K_0-groups.

2020 Mathematics Subject Classification: 46L05, 46L35

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Boundary value problem for nonlocal half-order of loaded ordinary linear differential equation

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Abstract: This study is devoted to finding a solution to a nonlocal conditional problem for a loaded fractal equation of half-order.

Let $Q[a,b] = \{0 < a < x < b\} \subset R_1$ be a rectangular region defined in Euclidean space. In $Q[a,b]$ the following problem

$$D_{b-}^{\frac{1}{2}} y(x) - py(x) + q_a y(a) + q_b y(b) = f(x),\ 0 < a < x < b, \tag{1}$$

$$\alpha y(a) + \beta y(b) = 0, \tag{2}$$

is considered. Here p, q_a, q_b, α and β are given real constants and

$$D_{b-}^{\frac{1}{2}} y(x) = -\frac{d}{dx}\int_x^b \frac{(t-x)^{-\frac{1}{2}}}{(-\frac{1}{2})!} y(t)dt.$$

Using the fundamental solution of the equation without loads, the so-called as a main relation consisting of two part was obtained. The first of which expression gives an arbitrary solution to equation (1), and the second part gives the necessary conditions that must be satisfied between this solution and the values that the solution takes on the boundary. Finding the boundary values of the solution from the obtained necessary conditions and substituting them into the expression in the first part of the main relation, we obtain an expression for the main solution to problem (1), (2).

Keywords: Half-order of loaded fractal equation, fundamental solution, basic relation and necessary condition

2020 Mathematics Subject Classification: 34A08, 34B05

References:

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The second order absolute stable difference scheme for semi-linear delay parabolic differential equation with Robin condition

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Abstract: In this work, we establish the stability theorem on the semi-linear delay parabolic differential equation with Robin condition. The stable second order of accuracy difference scheme in t for the approximate solution of this problem is given. Numerical results for second order of accuracy difference scheme in t are presented and compared with the first order of accuracy difference scheme in t.

Keywords: Delay parabolic equation, stability, Robin condition

2020 Mathematics Subject Classification: 35K20, 65M06, 65M12

References:

[1] A. Ashyralyev, D. Agirseven, S.B. Mua'zu, A note on the delay nonlinear parabolic differential equations, Filomat, 38(16), 5761–5778, 2024.

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On positive solutions of the heat equation with a singular potential

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Abstract. In this paper we study the existence of nonnegative solutions of the first initial-value problem for a linear heat equation with a singular potential in the cylinder $\Omega\times(0,T)$, where $\Omega\subset R^n(n\geq 3)$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, containing the ball $B_\rho = B(0,\rho)$ of radius $\rho,\ \rho\leq 1$, centered at the origin of coordinates and $T>0$. We find an exact condition on the potential ensuring the existence of a nonnegative solution of that problem. In addition, the lower estimates for non-negative solutions of this problem is established.

Keywords: Nonnegative solution, heat equation, singular potential, existence, lower estimate.

In the cylinder $\Omega\times(O,T)$ is considered the problem of finding a non-negative function $u(x,t)$:

$$u_t-\Delta u-V(x)u=0,\quad (x,t)\in\Omega\times(0,T), \tag{1}$$

$$u(x,t)=0,\ \ x\in\partial\Omega,\ \ t\in(0,T), \tag{2}$$

$$u(x,o)=u_0(x),\quad x\in\Omega, \tag{3}$$

where $\Omega\subset R^n(n\geq 3)$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, containing the ball $B_\rho=B(0,\rho)$ of radius ρ $(\rho\leq 1)$ centered at the origin $O, x=(x_1,...,x_n)$ and Δ is a standard Laplacian and $T>0$.

By a solution of equation (1) we mean a nonnegative generalized function (distribution) $u\in D'(\Omega\times(0,T))$ such that $Vu\in L^1_{loc}(\Omega\times(0,T))$.

Condition (3) holds in the sense of

$$ess\lim_{t\to+0}\int\limits_{\Omega}u(x,t)\eta(x)dx=\int\limits_{\Omega}u_0(x)\eta(x)dx$$

for any function $\eta\in D(\Omega)=C_0^\infty(\Omega)$.

It is assumed that $0\leq V\in L^1_{loc}(\Omega)$ and $0\leq u_0\in L^2(\Omega)$.

Let us introduce the polar coordinates $(r,\omega), r=|x|, \omega=(\omega_1,\omega_2,\cdots,\omega_{n-1})$, centered at the origin. For a radial function the Laplace operator in polar coordinates is of the form

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{n-1}{r}\cdot\frac{\partial}{\partial r}.$$

Let $\varphi(x)>0$ be a radial function singular at zero O such that $\Delta\varphi\in L^1_{loc}(\Omega)$ and for any function $v\in C_0^1(\Omega)$ the weighted Sobolev estimate

$$\left(\int\limits_{\Omega}|v|^q\varphi^2dx\right)^{2/q}\leq Const\cdot\int\limits_{\Omega}\left(|\nabla v|^2+v^2\right)\varphi^2dx \tag{4}$$

holds for some $q>2$. Set

$$V_0(x)=-\frac{\Delta\varphi}{\varphi},\ x\in B_\rho,\ V(x)\in L^\infty(\Omega\setminus B_\rho).$$

In this paper is analyzed the behavior of nonnegative solutions of problem (1) - (3) in the neighborhood of zero O and is proved that if $0\leq V(x)\leq V_0(x)$ in B_ρ, then this problem has a non-negative solution for any nonnegative initial function $u_0\in L^2(\Omega)$ and is established lower estimates for non-negative solutions of this problem.

The main results of the paper is the following theorems:

Theorem 1. *Let the (measurable) potential $V(x)\geq 0$ satisfy $V(x)\in L^\infty(\Omega\setminus B_\rho)$ and $V_0(x)=-\Delta\varphi/\varphi$ in B_ρ, where the function $\varphi(x)>0$ such that $\Delta\varphi\in L^1_{loc}(\Omega)$ and the inequality (4) holds. If $0\leq V(x)\leq V_0(x)$ in B_ρ, then problem (1) - (3) has a nonnegative solution for any nonnegative initial function $u_0\in L^2(\Omega)$.*

Theorem 2. *Let $V(x)\geq V_0(x)$ and $u_0(x)>0$ in $\Omega\times(0,\varepsilon)$ for each $\varepsilon\in(0,T)$. Then given subdomain $0\in\Omega'\subset\subset\Omega$ there is a constant $C=C(\varepsilon,\Omega')>0$ such that for almost all $(x,t)\in\Omega'\times[\varepsilon,T)$ the following inequality holds*

$$u(x,t)\geq C\varphi(x)>0.$$

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Investigation of the solution of a boundary value problem with variable coefficients which principal part is Cauchy-Riemann equation

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Abstract: Let D be a convex bounded domain in the direction of x_2, $\partial D = \Gamma$ is a Lyapunov curve. Consider the following problem in the domain D:

$$u_{x_2}(x) + \sqrt{-1}u_{x_1}(x) = a(x)u(x) + f(x),\ x \in D, \tag{1}$$

$$\alpha_1(x_1)u\left(x_1, \gamma_1(x_1)\right) + \alpha_2(x_1)u\left(x_1, \gamma_2(x_1)\right) = \varphi(x_1),\ x_1 \in [a_1, b_1]. \tag{2}$$

Here $a(x)$, $f(x)$, $\alpha_1(x_1)$, $\alpha_2(x_1)$ and $\varphi(x_1)$ are known continuously functions.

Using of the fundamental solution of the main part of equation (1), the basic expression consisting of a two-part is found. The first part of which gives an any solution of equation (1) in the domain D if $\xi \in D$, but the second part gives necessary conditions for relations between obtained solution with boundary conditions for $\xi \in \Gamma$.

Theorem 1. Let the D be a bounded convex domain in the x_2 direction and Γ is a Lyapunov curve, if $a(x)$, $f(x)$ are continuous, $\alpha_1(x_1)$, $\alpha_2(x_1)$ belong to the Hölder class, and $\varphi(x_1)$ satisfies the following conditions $\varphi(a_1) = \varphi(b_1) = 0$, $\varphi(x_1) \in C^{(1)}[a_1, b_1]$ then the solution of problem (1), (2 is a regular function and has Fredholm properties.

Keywords: Cauchy-Riemann equation, nonlocal boundary condition, basic relation, necessary conditions, Fredholm property

2020 Mathematics Subject Classification: 35J67

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Toward a new phase of development of the theory differential and integral

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Abstract: In this work, the new approach for concept to the as multiplicative derivative as well of multiplicative integral are proposed as follows

(1) $$f^{[']}(x) = \lim_{h\to 0} \sqrt[h]{\frac{f(x+h)}{f(x)}}, \qquad \lim_{\substack{|\Delta x_k|\to 0,\\ n\to\infty}} \prod_{k=1}^{n} f(x_k)^{\Delta x_k} = \int_a^{\int^b} f(x)^{dx}.$$

This terminology was first used by Volterra. The discrete analogue of these new operations made it possible to create an axiomatic theory of many problems of discrete mathematics. For example, the Fibonacci sequence $p_{n+2} = p_n + p_{n+1},\ n \geq 0,\ p_0 = \alpha,\ p_1 = \beta$ in discrete mathematics, in terms of the proposed additive derivative is reduced to the Cauchy problem $p_n^{['']} + p_n^{[']} - p_n = 0,\ n \geq 0,\ p_0 = \alpha,\ p_1 = \beta$. Further by using the concept of exponentiation from the left side the concept of powerative integral is obtained as

(2) $$\int_l^{\swarrow \int^{x_0} dx} f(x) = \lim_{\substack{|\Delta x_k|\to 0,\\ n\to\infty}} {}^{\Delta x_n} f(x_n)^{{}^{\Delta x_{n-1}} f(x_{n-1})^{\cdots {}^{\Delta x_1} f(x_1)^{{}^{\Delta x_0} f(x_0)}}}.$$

Here, exponentiation from the left side is defined as ${}^m x^{\swarrow} = x^{x^{x^{\cdot^{\cdot^{x}}}}} \swarrow$ that is calculation draw from above to down.

The inverse of operation of the powerative integral, that is powerative derivative, is expressed as the root of the root with from left side form class continuous functions is designated as the follows

(3) $$\lim_{h\to 0} \sqrt[h\swarrow]{\sqrt[f(x)]{f(x+h)}} = f^{\{'\}}(x).$$

Keywords: Discrete mathematics, multiplicative derivative and integral, poweratrive derivative and integral

2020 Mathematics Subject Classification: 11A67

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[1] N.A. Aliyev, Y.Y. Mustafayeva, From Archimedes to Powerative Integral, In: Proceeding of IAM, vol. 11, no 2, 103–112, 2022.

Itô-Malliavin type equations and application to finance

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Abstract: Let X_u be an unknown $\mathcal{F}_u$ adapted Gaussian process and $X_u \in \mathbb{D}_{1,2}$. Let F be a known function with respect to the time t, u, the $\mathcal{F}_u$ adapted Wiener processes W_u and the process X_u. Additionally, F is continuously differentiable with respect to t, u and second-order continuously differentiable with respect to W_u, X_u. Let D_t be the Malliavin derivative operator. Then, equations which have the form of

$$D_t X_u = F\left(t, u, W_u, X_u\right) \cdot 1_{u>t}$$

are called Itô-Malliavin type equations. Notice that $u > t$ is the only case considered here since $D_t X_u = 0$ for all $\mathcal{F}_u$ adapted Gaussian processes X_u when $u < t$.

In this paper, we introduce a special type of SDE (stochastic differential equation) driven by the Malliavin derivative operator and the Itô integral, which are called Itô-Malliavin type equations. Four types of Itô-Malliavin type equations and a corresponding Stratonovich-Malliavin type equation are solved here by some general methods. We also illustrate some specific examples of every Itô-Malliavin type equation such as the geometric Brownian motion, the Ornstein-Uhlenbeck process, the Brownian bridge, and the Bessel process. Some typical properties of these solutions are shown as well. Furthermore, a higher order Itô-Malliavin type equation is discussed as an extension of the Itô-Malliavin type equation. Applications to finance, such as computations of price sensitivities and hedging, are also considered.

Keywords: Stroock lemma, stochastic differential equations, Itô-Malliavin type equations, Stratonovich-Malliavin type equations, geometric Brownian motion; Ornstein-Uhlenbeck process, Brownian bridge, Bessel process

2020 Mathematics Subject Classification: 60H05, 60H07, 60H10, 91G20

Asymptotic distribution of limit theorems for kernel regression estimator for quasi-associated functional censored time series within single index structure

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Abstract: In this paper, we develop kernel-based estimators for regression functions under a functional single-index model, applied to censored time series data. By capitalizing on the single-index structure, we reduce the dimensionality of the covariate-response relation ship, thereby preserving the ability to capture intricate dependencies while maintaining a relatively parsimonious form. Specifically, our framework utilizes nonparametric kernel estimation within a quasi-association setting to characterize the underlying relationships. Under mild regularity conditions, we demonstrate that these estimators attain asymptotic normality

Keywords:kernel regression estimation; weak dependence data; quasi-associated variables; single functional index model.

MSC:62G20; 62G05; 62G32; 62G08; 62G35; 62G07; 62E20

Numerical Solution of semi-linear delay parabolic differential equation with Robin condition

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Abstract: In this work, we consider the well-posedness theorem on the semi-linear delay parabolic differential equation with Robin condition. The stable difference scheme for the approximate solution of this problem is presented. Numerical results are given.

Keywords: Delay parabolic equation, stability, Robin condition

2020 Mathematics Subject Classification: 35K20, 65M06, 65M12

References:

[1] A. Ashyralyev, D. Agirseven, S.B. Mua'zu, A note on the delay nonlinear parabolic differential equations, Filomat, 38(16), 5761–5778, 2024.
[2] A. Ashyralyev, S.B. Mua'zu, Bounded solutions of semi-linear parabolic differential equations with unbounded delay terms, Mathematics, vol. 1, 5470 (14 pp.), 2023.
[3] A. Ashyralyev, D. Agirseven, B. Ceylan, Bounded solutions of delay nonlinear evolutionary equations, J. Comput. Appl. Math., vol. 318 69–78, 2017.

A remark on fractional spaces generated by the second order differential operator with dependent coefficient and periodic conditions

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Abstract: We consider the second order differential operator A^x defined by

$$A^x u = -\left(a(x)u_x(x)\right)_x + \sigma u(x), \sigma \geq 0, x \in \mathbb{R}$$

with domain

$$D = \left\{ u : u, u'' \in C(\mathbb{R}), u(x) = u(x+2\pi), x \in \mathbb{R}, \int_0^{2\pi} (a(x)u_x(x))_x dx = 0 \right\}.$$

Here, $a(x) = a(x+2\pi)$, $x \in \mathbb{R}$ and $a(x) \geq a_0 > 0$ is continuously differentiable function defined on $\mathbb{R}$.

We obtain the estimates for the Green's function. We also prove that for any $\alpha \in (0, \frac{1}{2})$, the norms in the spaces $E_\alpha = E_\alpha(\mathring{C}(\mathbb{R}), A^x)$ and $\mathring{C}^{2\alpha}(\mathbb{R})$ are equivalent. Furthermore, we prove the positivity of the operator A^x in Hölder spaces of $\mathring{C}^{2\alpha}(\mathbb{R}), \alpha \in (0, \frac{1}{2})$. In the applications, we establish theorems on well-posedness of local and nonlocal boundary value problems for elliptic equations in the Hölder spaces. (see, [1], [2]).

Keywords: Positivity of differential operators, periodic boundary conditions, boundary value problems, Green's function

2020 Mathematics Subject Classification: 35J08, 35J58, 47B65

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A note on interpolation spaces generated by the second oder differential operator with dependent coefficient and Samarskii-Ionkin conditions

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Abstract: In this study, the second order differential operator A^x defined by

$$A^x(u) = -(a(x)u_x)_x + \sigma u(x), \quad \sigma > 0, \ 0 < x < 1$$

with domain

$$u(0) = 0, u_x(0) = u_x(1) + \mu u(1),$$

where $\mu > 0$, $a(0) = a(1)$ and $a(x) \geqslant a_0 > 0$ is continuously differentiable function defined on $[0, 1]$.

Estimates for the Green's function are established. It is also proved that for each $\theta \in (0, \frac{1}{2})$, the interpolation space $\mathcal{B}_\theta(\mathring{C}[0, 1], A^x)$ and the Hölder space $\mathring{C}^{2\theta}[0, 1]$ are topologically equivalent. Moreover, the positivity of the operator A^x in Hölder spaces of $\mathring{C}^{2\theta}[0, 1], \theta \in (0, \frac{1}{2})$ is proved. In applications, the theorems on well-posedness of local and nonlocal boundary value problems for paroabolic equations in the Hölder spaces are established. (see, [1], [2]).

Keywords: Positivity of differential operators, periodic boundary conditions, boundary value problems, Green's function

2020 Mathematics Subject Classification: 35J08, 35J58, 47B65

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On the critical behavior for inhomogeneous semilinear biharmonic heat equations on exterior domains

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Abstract: This report is devoted to the existence and nonexistence of global weak solutions to the inhomogeneous semilinear heat equations with forcing terms on exterior domains

$$\begin{cases} u_t + \Delta^2 u = |u|^p + f(x), & \text{in } D^c \times (0,\infty), \\ u(x,0) = u_0(x), & \text{in } D^c, \end{cases} \tag{1}$$

where $p > 1$ is a constant, Δ is the Laplace operator, and $D = \overline{B_1}$ is the closed unit ball, and D^c is its complement in $\mathbb{R}^N$.

We investigate the critical behavior of solutions to the semilinear biharmonic heat equation with forcing term $f(x)$, under six homogeneous boundary conditions. By employing a method of test function, we derive the critical exponents p_{Crit} in the sense of Fujita. Moreover, we show that $p_{Crit} = \infty$ if $N = 2, 3, 4$ and $p_{Crit} = \frac{N}{N-4}$ if $N \geq 5$. The impact of the forcing term on the critical behavior of the problem is also of interest, and thus a second critical exponent in the sense of Lee-Ni, depending on the forcing term is introduced. We also discuss the case $f \equiv 0$, and present the finite-time blow-up results for the subcritical and critical cases.

Keywords: Exterior problem, Fujita critical exponent, nonexistence, existence

2020 Mathematics Subject Classification: 35A01, 35B09, 35B44

On the stable difference scheme for numerical solving reverse parabolic source identification problem

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Abstract The aim of current work is to study stability aspects of the Rothe difference scheme for approximate solving of source identification problem for reverse parabolic equation with the initial and nonlocal conditions

$$\begin{cases} \frac{dv(t)}{dt} - Av(t) = p + f(t), 0 < t < 1 \ , \\ v(0) = \varphi, \\ v(1) = \sum\limits_{k=1}^{l} \mu_k v(\gamma_k) + \psi \end{cases} \tag{1}$$

for given functiion $f : [0,1] \to H$ and elements φ, $\psi \in H$. Here $A : H \to H$ is a self-adjoint positive definite operator in an arbitrary Hilbert space H and γ_k, μ_k, $k = 1, ..., r$ are known real numbers such that

$$\sum_{k=1}^{r} |\mu_k| < 1, 0 \leq \gamma_1 < \gamma_2 < ... < \gamma_r < 1 \tag{2}$$

Wellposedness of source identification problem was established in [1]. Rothe difference scheme for the direct reverse parabolic problem with integral boundary condition was investigated in [2].

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Keywords: difference scheme, inverse problem, source identification problem, well-posedness, stability estimate, reverse parabolic equation

2020 Mathematics Subject Classification: 39A14, 65N06, 65J22

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On inhomogeneous exterior Robin problems with critical nonlinearities

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Abstract: The paper studies the large-time behavior of solutions to the Robin problem for PDEs with critical nonlinearities. For the considered problems, nonexistence results are obtained, which complements the interesting recent results by Ikeda et al. [1], where critical cases were left open. Moreover, our results provide partially answers to some other open questions previously posed by Zhang [2].

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Keywords: inhomogeneous Robin problem, exterior domain, large-time behavior of solutions.

2020 Mathematics Subject Classification: 35K70, 35A01, 35B44

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A dynamical systems approach to inflammation-driven hematopoietic disruption: feedback mechanisms and malignant transition

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Abstract: Chronic inflammation disrupts hematopoietic homeostasis, promoting aberrant myelopoiesis and clonal expansion of mutated stem cells. To unravel the interplay between local (intrinsic bone marrow) and global (systemic) inflammatory feedback in this process, we propose a mathematical model formulated as a system of nonlinear differential equations:

$$\frac{dS}{dt} = (2p_0 - 1)Sv_0 \tag{1}$$

$$\frac{dP}{dt} = 2(1 - p_0)Sv_0 + (2p_1 - 1)v_1 P \tag{2}$$

$$\frac{dD}{dt} = 2(1 - p_1)Pv_1 - d(I)D \tag{3}$$

$$\frac{dI}{dt} = ad(I)D - d_1 I \tag{4}$$

$$\frac{dI_{BM}}{dt} = \beta I - d_2 I_{BM} \tag{5}$$

This framework captures the nonlinear dynamics of stem cell self-renewal (S), progenitor proliferation (P), and inflammatory signaling (I,$I_B M$), enabling classification of healthy, myelodysplastic, and leukemic states through phase-space analysis. We demonstrate that global feedback enhances hematopoietic resilience at moderate levels, whereas chronic inflammation destabilizes the system by biasing progenitor differentiation, fostering clonal dominance. Sensitivity analysis reveals critical thresholds at which feedback loops drive transitions between physiological and pathological regimes, illustrating how mutated clones exploit inflammatory niches.

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Keywords: Non-linear dynamics, Phase space Analysis, Sensitivity Analysis.

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Mittag-Leffler function constrained quasi-synchronization for fractional-order impulsive neural networks

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Abstract: We develop a general framework for quasi-synchronization in chaotic fractional-order neural networks using impulsive state-feedback controllers that incorporate both real-time and delayed error terms. These controllers are applied at fixed impulsive moments to drive the synchronization error into an ϵ-neighborhood of the origin. By combining the fractional Halanay inequality, a generalized Gronwall bound, and Lyapunov methods, we derive Mittag-Leffler function-constrained conditions that guarantee bounded error convergence in several FONN architectures—recurrent, inertial, and reaction-diffusion. These results are supported by numerical simulations conducted for the corresponding FONN models. Finally, we apply the proposed quasi-synchronization schemes—across various FONN models and control strategies—to an image encryption task, evaluating their performance under a common chaotic encoding algorithm. The results are contrasted with those obtained under finite-time synchronization, revealing critical differences in synchronization behavior and their practical implications for secure image transmission.

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Keywords: Fractional-order neural networks, quasi-synchronization, impulsive control, Lyapunov function approach, Mittag-Leffler function, image encryption.

2020 Mathematics Subject Classification: 34A08, 93D05, 35R11, 93C95, 94A08

The quadratic $\mathcal{B}$-spline method for approximating a system of Volterra integro fractional-differential equations utilizing both classical and fractional derivatives

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Abstract: The quadratic $\mathcal{B}$-spline method is a widely recognized numerical technique for solving systems of Volterra integro-differential equations that involve both classical and fractional derivatives (SVIDE's-CF). This study presents an improved application of the quadratic $\mathcal{B}$-spline approach to achieve highly accurate and computationally efficient solutions. In the method developed in this paper, control points are treated as unknown variables within the framework of the approximate solution. The fractional derivative ${}^{C}_{a}\mathcal{D}^{\sigma}_{x}$ is considered in the Caputo sense. First, we divide the domain into subintervals, then construct quadratic $\mathcal{B}$-spline basis functions over each sub-interval. The approximate solution is presented as a quadratic combination of these $\mathcal{B}$-spline functions over each sub-interval, where the control points act as variables. To simplify the system of (VIDE's-CF) into a solvable set of algebraic equations, the collocation method is applied by discretizing the equations at chosen points within each subinterval. The Jacobian matrix method is employed to perform computations efficiently.In addition, a careful, step-by-step algorithm for employing the proposed method is presented to simplify its use, we implemented the method in a Python program and optimized it for efficiency. Experimental example effectiveness and accuracy of the proposed technique and its comparison with present techniques in terms of accuracy and computational efficiency. This study presents an approximate method for solving the linear system associated with Volterra integro-differential equations, encompassing classical and fractional orders (LSVIDE's-CF). For the derivation, it deals with quadratic $\mathcal{B}$-spline interpolation functions. Which takes the following general forms:

$$\begin{aligned}\mathcal{P}_i(x)\mathcal{U}_i''(x) + a_{i0}(x)\,{}^{C}_{a}\mathcal{D}^{\sigma_{i0}}_{x}\mathcal{U}_i(x) + a_{i1}(x)\,{}^{C}_{a}\mathcal{D}^{\sigma_{i1}}_{x}\mathcal{U}_i(x) + a_{i2}(x)\mathcal{U}_i(x)\\ = \mathcal{F}_i(x) + \sum_{j=0}^{m}\omega_{ij}\int_a^x \mathcal{K}_{ij}(x,s)\,{}^{C}_{a}\mathcal{D}^{\beta_{ij}}_{s}\mathcal{U}_j(s)\,ds.\end{aligned} \tag{1}$$

Under the following conditions:

$$\left[\mathcal{D}^{k_i}_{x}\mathcal{U}_i(x)\right]_{x=a} = \vartheta_{ik_i}, \forall k_i = 0,1,\ldots,\mu_i - 1, \text{ and } i = 0,1,\ldots,m. \tag{2}$$

The variable coefficients $\mathcal{P}_i(x)(\not\equiv 0)$, $a_{i0}(x)$ and $a_{i1}(x) \in C([a,b],\mathbb{R})$ and $\mathcal{K}_{ij} \in C(\Theta,\mathbb{R})$, $\Theta = \{(x,s) : a \le x < s \le b\}$, for each $i,j = 0,1,\ldots,m$, with fractional orders: $\sigma_{i1} > \sigma_{i0} > 0$ and $\beta_{im} > \beta_{i(m-1)} > \cdots > \beta_{i1} > \beta_{i0} = 0$, and for all $i,j = 0,1,\ldots,m$. Furthermore, the $\mu_i = \max\left\{2, m^{\beta}_{im}\right\}$ for all $i = 0,1,\ldots,m$, where $m^{\beta}_{ij} - 1 < \beta_{ij} \le m^{\beta}_{ij}$, $m^{\beta}_{ij} = \lceil\beta_{ij}\rceil$, $\omega_{ij} \in \mathbb{R}$ also for all $i,j = 0,1,\ldots,m$.

Keywords: Volterra integro-fractional differential equation (VIDE's), Quadratic $\mathcal{B}$-spline functions, Caputo fractional derivative, Collocation method, Jacobian matrix algorithm, Clenshaw-Curtis quadrature rule.

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Nonparametric estimation of distribution function: two-measurement problem

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Abstract: We propose a new plug-in estimator for the distribution function of two independent but heterogeneously distributed random variables, where one has a density and the other only has a distribution. Contrary to the extant literature, no restrictive assumption is imposed on the distribution function, and only mild smoothness conditions are imposed on the density. We show that the proposed estimator is asymptotically unbiased. Our work has broad applications, in particular in medicine, where for a number of variables of interest, it is often advisable to construct values that are averages of two independently obtained measurements.

Previous results include W. Kordecki, A. Saavedra and R. Cao, G. E. Willmot, G. E. and J-K Woo, C. Chesnau, F. Comte and F. Navarro, K. S. Trivedi, N. N. Midhu, I. Dewan, K. K. Sudheesh and E. P. Sreedevi. Some techniques from [1] are used.

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Keywords: Distribution function, density, nonparametric estimation, two-measurement problem

2020 Mathematics Subject Classification: 62G05, 62G07, 62G20

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On the spectral stability of the generalized biharmonic Steklov problem

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Abstract: This report is devoted to study the spectral stability of a natural biharmonic Steklov problem when the domain undergoes perturbations. We establish sharp conditions on boundary variations that guarantee the stability of both eigenvalues and eigenfunctions.

To demonstrate the sharpness of these conditions, we explore alternative types of boundary perturbations which result either in spectral degeneration or in the emergence of a strange term in the limiting problem. Specifically, we examine these effects in the context of a boundary homogenization problem, which displays a trichotomy in its asymptotic behavior.

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP26194963).

Keywords: Steklov boundary conditions, multi-parameter eigenvalue problems, biharmonic Steklov eigenvalues, domain perturbations, spectral stability

2020 Mathematics Subject Classification: 35J40, 35B20, 35P15

Examples of 3D gl-regular Nijenhuis operators

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Abstract: Nijenhuis operator fields with vanishing Nijenhuis torsion naturally appear in various areas of differential geometry, algebra, and mathematical physics. They often encode compatibility conditions for systems of partial differential equations. Among such structures, a special role is played by gl-regular (algebraically stable) Nijenhuis operators, whose classification forms a central problem in the theory of Nijenhuis Geometry.

A complete classification of gl-regular Nijenhuis operators in dimension two was obtained by A. Bolsinov, V. Matveev, and A. Konyaev. We also arrived at this classification independently while studying quadratic integrals of geodesic flows of pseudo-Riemannian metrics.

This talk will present examples of three-dimensional gl-regular Nijenhuis operators and discuss the first steps toward their classification.

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Keywords: Nijenhuis tensor, Nijenhuis torsion, gl-regular operators.

2020 Mathematics Subject Classification: 53B30, 53B50

Analysis of a double nonlinear cross-wise diffusion system

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Abstract: This paper is devoted to a double nonlinear cross-wise diffusion system with the Cauchy problem in the area $Q = \{(x,t) \mid x \in \mathbb{R}^N,\ t \in \mathbb{R}_+\}$:

$$(1)\qquad \begin{cases} \rho_1(x)\partial_t u = u^{q_1} v^{p_1} div\left(u^{m_1-1}|\nabla u|^{p-2}\nabla u\right) + \rho_1(x)\gamma_1(t)v^{\beta_1} \\ \rho_1(x)\partial_t v = u^{q_2} v^{p_2} div\left(v^{m_2-1}|\nabla v|^{p-2}\nabla v\right) + \rho_1(x)\gamma_2(t)u^{\beta_2} \end{cases},$$

$$(2)\qquad \begin{cases} u(x,0) = u_0(x) \geq 0 \\ v(x,0) = v_0(x) \geq 0 \end{cases},\ x \in \mathbb{R}^N.$$

Here, $q_i, p_i \neq 1, m_i > 1, p \geq 2, \beta_i > 1, \rho_1(x) = |x|^{-n_1}, n_1 > 0, \gamma_i(t) = (Ct + C_0)^{-l_i}, C > 0, C_0 \geq 0, l_i \in \mathbb{R}, i = 1,2$ – are given numerical parameters.

We are interested in compactly supported self-similar solutions satisfying (1)-(2), in the distribution sense. Using self-similar analysis [1], we constructed the self-similar solution with the Barenblatt profile. Throughout this paper we mainly use techniques outlined in the work [2]-[3].

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Keywords: Global solution, weak solution, estimate solution, asymptotic behaviour

2020 Mathematics Subject Classification: 35B40, 35C06, 35K40.

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Integral representations of solutions of coercive solvable problems of the Gellerstedt equation

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Abstract: In domain $\Omega \subset R^2$, bounded at $y > 0$ by smooth curve σ and by segment $AB : y = 0, 0 \le x \le 1$, we consider the degenerate Gellerstedt equation

$$L_G u \equiv y^m u_{xx} + u_{yy} = f(x, y). \tag{1}$$

Theorem 1. *For any regular boundary value problem for the degenerate elliptic Gellerstedt equation there are such functions $\overline{\mu}$ and $\overline{\mu}_0$ that the solution of the problem is expressed by the formula*

$$u(x,y) = L_G^{-1} f = \int_\Omega \varepsilon(x,y,\xi,\eta) f(\xi,\eta) d\xi\eta +$$

$$+ \int_\Omega \left(\int_0^l \frac{\partial \varepsilon}{\partial n_\varepsilon} \overline{\mu} ds \right) f(\xi,\eta) d\xi\eta + \int_\Omega \overline{\mu}_0 f(\xi,\eta) d\xi\eta. \tag{2}$$

Here

$$\varepsilon(x,y,\xi,\eta) = k \cdot \left(r_1^2\right)^{-\beta} (1-\sigma)^{1-2\beta} F(1-\beta, 1-\beta; 2-2\beta; 1-\sigma)$$

- a fundamental solution of (1) equation,

$$\beta = \frac{1}{2(m+2)}, \quad k = \left(\frac{4}{m+2}\right)^{4\beta-2} \cdot \frac{\Gamma(\beta)}{\Gamma(1-\beta)\cdot\Gamma(2\beta)}, \quad \sigma = \frac{r^2}{r_1^2},$$

$$r^2 = (x-\xi)^2 + \frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}} - \eta^{\frac{m+2}{2}}\right)^2,$$

$$r_1^2 = (x-\xi)^2 + \frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}} + \eta^{\frac{m+2}{2}}\right)^2,$$

$F(a,b;c;z)$ - hypergeometric function; $\Gamma(z)$ - gamma-function.

The criterion (necessary and sufficient condition) that the corresponding problem is a boundary value problem or problem with inner-boundary conditions have been also established.

The proof is based on the properties of the potentials of the simple and double layers described above, the properties of special functions, Green's formula and Riesz's theorem.

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Keywords: integral representation, Gellerstedt equation, fundamental solution, boundary value problem, inner-boundary condition, hypergeometric function

2020 Mathematics Subject Classification: 35L80, 35M10, 35N30, 33C05

On the inverse problem for a fractional pseudo-parabolic equation with involution

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Abstract: In this work, an inverse problem for the fractional analog of a parabolic equation with involution is studied, focusing on determining the solution and the unknown right-hand side depending on the spatial variable. The problem with initial and boundary Dirichlet conditions, as well as an overdetermination condition, is examined. The investigated problem is analyzed using the Fourier method. Eigenfunctions and eigenvalues of the spectral problem with Dirichlet conditions for the non-local analog of the Laplace operator are found. Theorems on the existence and uniqueness of solutions to the considered problem are proved.

Similar problems in the case $n = 1$ were studied in [1].

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Keywords: fractional pseudo-parabolic equation, Caputo operator, inverse problem, involution

2020 Mathematics Subject Classification: 35R30, 35K20, 35J25

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On one method of constructing a fundamental system in the space of solenoidal functions in a multidimensional rectangular parallelepiped

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Abstract: In the space of solenoidal functions for domains represented by a multidimensional rectangular parallelepiped, in particular, a square and a cube, an orthogonal fundamental system is constructed. We use the concept of a fundamental system in the sence of Ladyzhenskaya [1]. Throughout the report, $\Omega = (0, l)^d \subset \mathbb{R}^d$, $d \geq 2$. We propose to consider the following spectral problem:

$$\sum_{k=1}^{d} \partial_{x_k}^4 U(x) = \lambda^2(-\Delta)U(x),\ x \in \Omega; \quad U(x) = 0,\ \partial_{\vec{n}} U(x) = 0,\ x \in \partial\Omega. \tag{1}$$

We use the following result established in papers [2]– [3]:

Theorem 1.1. *The set of generalized eigenfunctions $\{u_n(x),\ n \in \mathbb{N}\}$ of the spectral problem* (1) *belongs to the space $\overset{\circ}{W}{}_2^2(\Omega)$ and forms an orthogonal basis for the space $\overset{\circ}{W}{}_2^1(\Omega)$. Moreover, all eigenvalues $\{\lambda_n^2\}_{n\in\mathbb{N}}$ lie on the positive real half-axis, and the smallest eigenvalue λ_1^2 is strictly positive.*

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Keywords: space of solenoidal functions, fundamental system, eigenvalues, eigenfunctions, spectral problem, curl operator, rectangular parallelepiped

2020 Mathematics Subject Classification: 35J05, 35Q30, 76D05, 76D07

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To construct an adjoint differentiation operator at integral perturbation of a boundary condition of a boundary condition with a spectral parameter

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Abstract:

Problem statement

A spectral problem for a first-order differential equation with a spectral parameter in boundary conditions with an integral perturbation of the boundary condition is considered.

(1) $$l(u) \equiv u'(t) = \lambda u(t), \quad 0 < x < 1,$$

(2) $$U_1 \equiv u(0) - \alpha u(1) + \lambda \{u(0) - \beta u(1)\} = \int_0^1 \overline{p(x)} u(x) dx,$$

where $\alpha \neq 0$ and $\beta \neq 0$ are given complex numbers, $p(x) \in L_2(0,1)$.

Construction of the unperturbed adjoint operator

$$L\vec{y} = L \begin{pmatrix} y_1(x) \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1'(x) \\ y_1(0) - \alpha y_1(1) \end{pmatrix}$$

$$D(L) = \left\{ \vec{y} = \begin{pmatrix} y_1(x) \\ y_2 \end{pmatrix} \in H, \quad y_1 \in W_2^1(0,1), \quad y_1(0) - \beta y_1(1) - y_2 = 0 \right\}$$

$$L^* \vec{v} = L^* \begin{pmatrix} v_1(x) \\ v_2 \end{pmatrix} = \begin{pmatrix} -v_1'(x) \\ \frac{1}{1-\bar{\beta}} \left[v_1(1) - v_1(0) \right] + \frac{1-\bar{\alpha}}{1-\bar{\beta}} v_2 \end{pmatrix}$$

$$D(L^*) = \left\{ \vec{v} \in H \,:\, v_1(x) \in W_2^1(0,1), \quad \bar{\beta}\, v_1(0) - v_1(1) + (\bar{\alpha} - \bar{\beta}) v_2 = 0 \right\}$$

We have proved that $\langle L\vec{y}, \vec{v} \rangle_H - \langle \vec{y}, L^* \vec{v} \rangle_H = 0$, for $\vec{y} \in D(L)$, $\vec{v} \in D(L^*)$.

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Keywords: adjoint operator, first-order differential equation, spectral problem, integral perturbation of boundary condition

2020 Mathematics Subject Classification: 45A47

Asymptotic formulas and uniform difference schemes for solving hyperbolic perturbation problems with nonlocal conditions

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Abstract: The abstract nonlocal boundary value problem

$$\begin{cases} \varepsilon^2 u^{''}(t) + Au(t) = f(t), 0 < t < T, \\ u(0) = \alpha u(T) + \varphi,\ u^{'}(0) = \beta u^{'}(T) + \psi \end{cases}$$

for hyperbolic equations in a Hilbert space H with the self adjoint positive definite operator A and with an arbitrary $\varepsilon \in (0, \infty)$ parameter multiplying the derivative term is considered. An asymptotic formula for the solution of this problem with a small ε parameter is established. The high order of accuracy two-step uniform difference schemes for the solution of this problem are presented. The convergence estimates for the solution of these difference schemes are established.

Keywords: Hyperbolic perturbation problems, asymptotic formulas, uniform difference schemes.

2020 Mathematics Subject Classification: 35L90, 47D09, 34E10, 65M06

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On the boundedness of the convolution operator in anisotropic Morrey-type spaces

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Let $k \in \mathbb{Z}$, $G_k = [0, 2^k)^n + 2^k m$, $m \in \mathbb{Z}^n$. $\mathbb{G} = \bigcup_{k\in\mathbb{Z}} G_k$, $Q \in G_k$. The set of mutually disjoint cubes $\mathbb{T} = \{Q\} \subset \mathbb{G}$ is called a local partition of the space $\mathbb{R}^n$ if $\mathbb{R}^n = \overline{\bigsqcup_{Q\in\mathbb{T}} Q}$ and $|\mathbb{T} \cap G_k| < \infty$.

Let $\bar{n} = (n_1, ..., n_d)$: $n_i \in \mathbb{N}$, $|n| = n_1 + \cdots + n_d$, $\bar{k} = (k_1, ..., k_d)$: $k_i \in \mathbb{Z}$. Denote $G_{\bar{k}} = \{Q = Q_1 \times \cdots \times Q_d \; : \; Q_i \subset G_{k_i}, \, i = 1, ..., d\}$. Mutually disjoint cubes $\mathbb{T}_i = \{Q_i\} \subset G_{k_i}$ are called local partitions of the space $\mathbb{R}^{n_i}$, the set $\mathbb{T}_1, \ldots, \mathbb{T}_d$ - local partitions of the spaces $\mathbb{R}^{n_1}, \ldots, \mathbb{R}^{n_d}$ respectively. Family of mutually non-intersecting parallelepipeds $\mathbb{T} = \mathbb{T}_1 \times \cdots \times \mathbb{T}_d = \{Q = Q_1 \times \cdots \times Q_d \; : \; Q_i \subset \mathbb{T}_i, \, i = 1, ..., d\}$ will be called a local partition of the space $\mathbb{R}^{|\bar{n}|}$.

Let $\bar{p} = (p_1, ..., p_d)$, $\bar{q} = (q_1, ..., q_d)$, $\bar{\lambda} = (\lambda_1, ..., \lambda_d)$ such that $0 < p_i \le \infty$, $0 < q_i \le \infty$, $-\infty < \lambda_i < \infty$. We define an anisotropic local Morrey space $LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})$ as the set of measurable functions f for which

$$\|f\|_{LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})} = \left(\sum_{k_d\in\mathbb{Z}} \cdots \left(\sum_{k_1\in\mathbb{Z}} \left(2^{-\langle \bar{k}, \bar{\lambda}\rangle} \sum_{Q\in\mathbb{T}_{\bar{k}}} \|f\|_{L_{\bar{p}}(Q)} \right)^{q_1} \right)^{\frac{q_2}{q_1}} \cdots \right)^{\frac{1}{q_d}} < \infty.$$

The anisotropic classical Morrey space $M^{\bar{\lambda}}_{\bar{p}}$ is the set of Lebesgue measurable functions $f \in L^{loc}_{\bar{p}}(\mathbb{R}^{\bar{n}})$ for which

$$\|f\|_{M^{\bar{\lambda}}_{\bar{p}}} = \sum_{k_d\in\mathbb{Z}} \cdots \sum_{k_1\in\mathbb{Z}} \left(2^{\langle -\bar{k}, \bar{\lambda}\rangle} \sup_{Q\in G_{\bar{k}}} \|f\|_{L_{\bar{p}}(Q)} \right) < \infty,$$

here $\langle \bar{k}, \bar{\lambda}\rangle = k_1\lambda_1 + \cdots + k_d\lambda_d$.

Consider the convolution operator

$$(Tf)(x) = (K * f)(x) = \int_{\mathbb{R}^d} K(x-y) f(y) dy,$$

acting from one Morrey space to another Morrey space.

Theorem. Let $\mathbb{T}$ be some local partition of the space $\mathbb{R}^{|n|}$. Let $0 < \max(\bar{q}, 1) \le p \le \infty$, $0 < \bar{\lambda} < \frac{\bar{n}}{\bar{q}}$ and $0 \le \bar{\gamma} \le \frac{\bar{n}}{p}$, $0 < \bar{\alpha} = \bar{\gamma} - \bar{\lambda} + \frac{\bar{n}}{\bar{q}} < \frac{\bar{n}}{p}$.

If $f \in M^{\bar{\gamma}}_{p}$ and $g \in LM^{-\bar{\alpha}}_{p',\bar{\infty}}(\mathbb{T})$, then $f * g \in M^{\bar{\lambda}}_{\bar{q}}$ and the inequality

$$\|f * g\|_{M^{\bar{\lambda}}_{\bar{q}}} < c\|f\|_{M^{\bar{\gamma}}_{p}} \|g\|_{LM^{-\bar{\alpha}}_{p',\bar{\infty}}(\mathbb{T})},$$

where the constant c depends only on the parameters $\bar{n}$, $\bar{\lambda}$, $\bar{q}, \bar{\alpha}$, p.

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On an integral representation of the solution for the Neumann problem for the Laplace equation

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Abstract: On a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$, we consider the Neumann problem for the Laplace equation:

$$-\Delta_x u = 0, \quad x \in \Omega. \tag{1}$$

$$\left.\frac{\partial u}{\partial n_x}\right|_{x\in\partial\Omega} = \nu(x). \tag{2}$$

It is well known that the problem (1)–(2) is solvable if and only if

$$\int_{\partial\Omega} \nu(x)\, dS_x = 0, \quad \nu(x) = \frac{\partial u}{\partial n_x}. \tag{3}$$

It is natural to ask under what condition the solution of the problem (1)–(2) can be represented in an exact form.

The following holds:

Theorem 1.2. *Let*

$$\nu(x) = -\frac{\mu(x)}{2} + \int_{\partial\Omega} \frac{\partial\varepsilon_n(x,y)}{\partial n_y}\mu(y)\, dS_y, \quad \mu(x) \in C^{1+\alpha}(\partial\Omega), \tag{4}$$

and

$$\int_{\partial\Omega} \left(-\frac{1}{2} + \int_{\partial\Omega} \frac{\partial\varepsilon_n(x,y)}{\partial n_y}\, dS_y\right)\mu(x)\, dS_x = 0, \tag{5}$$

then the solution of the problem (1)–(2) *has the following form:*

$$u(x) = \int_{\partial\Omega} \varepsilon_n(x,y)\mu(y)\, dS_y, \tag{6}$$

where

$$\varepsilon_n(x,y) = \begin{cases} -\frac{1}{2\pi}\ln|x-y|, & n = 2, \\ \frac{1}{\omega_n(n-2)}\frac{1}{|x-y|^{n-2}}, & n \geq 3. \end{cases} \tag{7}$$

Remark 1.3. *The Theorem above is also valid for the general uniform elliptic equation*

$$Lu = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}(x)\frac{\partial u}{\partial x_i}\right) + c(x)u = 0, \tag{8}$$

where

$$a_{ij} = a_{ji}; \quad \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \delta\sum_{i=1}^{n}\xi_i^2, \quad \forall\xi =\in R^n, \quad \delta > 0.$$

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Keywords: Neumann problem, Laplace equation, integral representation, Green's function.

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Solvability of boundary value problems for differential equations with involution in Hilbert space

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Abstract: In a real Hilbert space H with norm $\|.\|$ consider the problem

(1) $$-y''(x) + \alpha y''(-x) + \rho^2 y(x) + f(x, y(x), y(-x)) = h(x), \; -1 < x < 1,$$

(2) $$y(-1) = 0, \; y(1) = 0.$$

Here $y(x)$ is the unknown function with values in a real Hilbert space H and $h(x)$: $(-1, 1) \to$H, $f(x, y, z) : (-1, 1) \times H \times H \to H$, $\rho > 0$, $-1 < \alpha < 1$. The derivative $y''(x)$ is understood as the limit in the norm of H. If $H = R$, then the scalar homogeneous problem

$$-y''(x) + \alpha y''(-x) + \rho^2 y(x) = 0$$

has the Green function.

Theorem. Let
1) $f(x, y, z) : (-1, 1) \times H \times H \to H$ be completely continuous.
2) There exist real positive numbers a, b, $a + b < \rho^2$ such that

$$(f(x, y, z), y) \geq -a\|y\|^2 - b\|y\|\|z\|$$

for all $(x, y, z) \in (-1, 1) \times$H$\times$H. Then the boundary value problem (1), (2) has at least one solution for every $h(x) \in L_1((-1, 1), H)$.

In the case $H = R$ the solvability theorems proved in other our work.

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Keywords: boundary value problems, eigenvalues, Hilbert space, involution perturbations, solvability.

2020 Mathematics Subject Classification: 34B27, 34G20, 34K10

On the solvability of the fundamental inverse problem for a system of first-order stochastic differential equations with degenerate diffusion

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Abstract: The problem of constructing a system of first-order Itô stochastic differential equations by the given properties of motion is considered.

Let the set

(1) $$\Lambda(t) : \lambda(x, y, t) = 0,\ \ \lambda \in R^m,\ \ \lambda = \lambda(x, y, t) \in C^{221}_{xyt}$$

be given. It is required to construct the equations of motion for the class of first-order Itô stochastic differential equations with degenerate diffusion

(2) $$\begin{cases} \dot{x} = f_1(x, y, t) \\ \dot{y} = f_2(x, y, t) + \sigma(x, y, t)\dot{\xi} \end{cases}$$

such that the set (1) is the integral manifold of system (2). Here $x \in R^{n_1}$, $y \in R^{n_2}$, $n_1+n_2 = n$, $\xi \in R^k$, and $\sigma(x, y, t)$ is a $(n \times k)$-matrix; $\{\xi_1(t, \omega), ..., \xi_k(t, \omega)\}$ is a system of independent Wiener processes [1] defined on some probability space .

Necessary and sufficient conditions for the existence of a given integral manifold of the constructed equation are obtained in terms of the equation's coefficients. These conditions are derived separately for two cases: when the manifold depends on all independent variables and when it depends only on a subset of them.

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Keywords: Stochastic differential equations, integral manifold, inverse problem, degenerate diffusion

2020 Mathematics Subject Classification: 60Gxx, 34A55

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Analysis and classification of fixed points of operators on a simplex

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Abstract: This report investigates the dynamical behavior of Lotka-Volterra type operators defined on the four-dimensional simplex, with a particular focus on fixed points and their structural representation via directed graphs (tournaments). Let the operator $V : S^4 \to S^4$ be defined by the system

(1) $$x'_k = x_k(1 + \sum_{i=1}^{5} a_{ki}x_i), \ k = 1, \ldots, 5,$$

where $a_{ki} = -a_{ik},\ x_i \geq 0,\ \sum_{i=1}^{5} x_i = 1$ and Δ_i are the fourth-order principal minors of the skew symmetric matrix $A = (a_{ij})$. We consider operators corresponding to tournaments preserving hamiltonian cycles of orders 3, 4, and 5. Using methods from algebraic graph theory, Youngs inequality, and Lyapunov function analysis, we establish explicit criteria for the existence and stability of interior fixed points. If $\Delta_1,\ \Delta_2,\ \Delta_3,\ \Delta_4 < 0$ and $\Delta_5 > 0$ then the operator has at least one internal fixed point $x^* \in int(S^4)$. If at least three of the values Δ_i are positive, then there are not internal fixed points.

Throughout this note we mainly use techniques from our work [1].

Keywords: Lotka-Volterra operator, simplex dynamics, fixed points, directed graphs, tournaments, cyclic structures, Lyapunov function.

2020 Mathematics Subject Classification: 37B25, 37C25

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Lotka–Volterra mappings on the simplex S^5 with sparse interactions: trajectory structure and attractors

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Abstract: This report investigates a class of discrete LotkaVolterra operators defined on a five-dimensional simplex, specifically focusing on those associated with interaction graphs featuring a limited number of connections: When the interaction follows a cyclic (ring) configuration

$$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 1$$

the mapping $V : S^5 \to S^5$ takes the form:

$$x_i' = x_i \left(1 + a_{i-1}x_{i-1} - a_i x_{i+1}\right), \quad i = 1, \ldots, 6,$$

where the indices are taken modulo 6, i.e.,

$$x_0 := x_6, \quad x_7 := x_1, \quad a_0 := a_6, \quad a_7 := a_1,$$

where $0 < a_1, \ldots, a_6 \leq 1$.

It is established that the sets $P = \{x \in S^5 : Ax \geq 0\}$ and $Q = \{x \in S^5 : Ax \leq 0\}$ are two-dimensional convex polyhedra in the simplex S^5, defined as the convex hulls of three points. Their interior (relative interior in S^5) is connected, and the sets P and Q themselves are homeomorphic to a triangle. This ensures a simple topological structure of the phase limit sets.

The mathematical findings provide a theoretical foundation for analyzing stability, survival strategies, and diversification processes in real-world biological systems.

Throughout this note, we mainly use techniques from our work [1].

Keywords: Discrete dynamical system, simplex, Lotka-Volterra mapping, Lyapunov functions, phase space, repellers, attractors, oriented graph, polyhedra, stability, signature dynamics, invariant sets

2020 Mathematics Subject Classification: 37B25, 37C25

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On an interpolation theorem for Lorentz spaces with mixed metric

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Abstract: This report is devoted to an interpolation theorem for Lorentz spaces with mixed metric.

Let $1 \leq \bar{q} = (q_1, q_2) \leq \infty$, $\bar{\varphi}(t) = (\varphi_1(t), \varphi_2(t)) \geq 0$. Let

$$\Lambda_{\bar{q}}(\bar{\varphi}) := \left\{ f : \left(\int_0^{+\infty} \left(\int_0^{+\infty} \left(f^{*_1 *_2}(t_1, t_2)\varphi_1(t_1)\varphi_2(t_2) \right)^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}} < \infty \right\},$$

where $f^{*_1 *_2} = f^{*_1 *_2}(t_1, t_2)$ is the nonincreasing permutation of a function f [1]. In the paper [2] were studied one-dimensional generalized Lorentz spaces.

Let $\delta > 0$ and $\varphi(t)$ be nonnegative function on $[0, +\infty)$. Let $C_\delta = \Big\{\varphi(t) :$ $\varphi(t)t^{-\delta}$ is an increasing function and $\varphi(t)t^{-1+\delta}$ is a decreasing function$\Big\}$. The class C is defined as follows: $C = \bigcup_{\delta > 0} C_\delta$.

Theorem 1.4. *Let $0 < \bar{p}_0 = (p_1^0, p_2^0) < \bar{p}_1 = (p_1^1, p_2^1) < \infty$, $0 < \bar{q} = (q_1, q_2) \leq \infty$, $\gamma_i = \frac{1}{p_i^0} - \frac{1}{p_i^1}$, $i = 1, 2$, $\varphi_1, \varphi_2 \in C$. Then the following inequality is true*

$$\left(L_{\bar{p}_0, \infty}, L_{\bar{p}_1, \infty} \right)_{\bar{\varphi}, \bar{q}} = \Lambda^{\bar{q}} \left(\bar{\psi} \right),$$

where $\bar{\psi}(t_1, t_2) = \left(\dfrac{t_1^{\frac{1}{p_0^1}}}{\varphi_1(t_1^{\gamma_1})}, \dfrac{t_2^{\frac{1}{p_0^2}}}{\varphi_2(t_2^{\gamma_2})} \right)$.

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Keywords: Lebesgue and Lorentz spaces, interpolation methods, interpolation theorem

2010 Mathematics Subject Classification: 46B70, 46E30

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O'Neil type inequalities in generalized local Morry type space

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Abstract: Let $\lambda \in \mathbb{R}$, $0 < p, q \leq \infty$, and $\mathbb{T} = \{Q\}$ be a local partition of $\mathbb{R}^n$. The local Morrey space was defined by Nursultanov and Suragan $LM^{\lambda}_{p,q}(\mathbb{T})$ as the set of measurable functions f for which

$$\|f\|_{LM^{\lambda}_{p,q}(\mathbb{T})} = \left(\sum_{k \in \mathbb{Z}} \left(2^{-k\lambda} \sum_{Q \in \mathbb{T}_k = \mathbb{T} \cap G_k} \|f\|_{L_p(Q)} \right)^q \right)^{\frac{1}{q}} < \infty.$$

This paper investigates norm estimates for convolution operators in generalized local Morrey-type spaces. Sufficient conditions for boundedness are established.

We prove Young-O'Neil-type inequalities in this setting, extending classical results to a more general framework. The obtained inequalities provide new estimates for convolution operators with kernels from weak Lebesgue spaces.

These results generalize known theorems for Lebesgue, Lorentz, and Morrey spaces, offering refined criteria for the boundedness of integral operators. Applications include potential operators and singular integrals in Morrey-type spaces.

Throughout this note we mainly use techniques from [1].

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Keywords: Morry space, convolution operator, Young-O'Neil's inequality, Riesz potential

2020 Mathematics Subject Classification: 47A05, 47B06

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O'Neil inequality in anisotropic grand Lorentz Spaces

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We obtain estimates for the norm of convolution operator in anisotropic grand Lorentz spaces. In these spaces, we prove O'Neil type inequalities and establish the boundedness of the logarithmic Riesz potential.

Theorem 1. Assume that $\overline{1} < \bar{p} < \bar{q} < \bar{\infty}, \overline{0} \leqslant \bar{\theta}, \bar{\theta}_0, \bar{s}, 1/\bar{r} = \overline{1} + 1/\bar{q} - 1/\bar{p}, \overline{0} < \bar{\tau} \leqslant \bar{\infty}$. If $\bar{\theta} = \bar{s} + \bar{\theta}_0$, then

$$\|f * g\|_{GL^{\bar{s}}_{\bar{q},\bar{\tau}}(\Omega)} \lesssim \|f\|_{GL^{-\bar{\theta}_0}_{\bar{p},\bar{\tau}}(\Omega)} \|g\|_{GL^{\bar{\theta}}_{\bar{r},\infty}(\Omega)}$$

$$\|f * g\|_{GL^{\bar{s}}_{\bar{q},\bar{\tau}}(\Omega)} \lesssim \|f\|_{GL^{\bar{\theta}_0}_{\bar{p},\bar{\tau}}(\Omega)} \|g\|_{GL^{\bar{\theta}}_{\bar{r},\infty}(\Omega)}$$

If $\bar{\theta} = \bar{s} - \bar{\theta}_0$, then

$$\|f * g\|_{GL^{\bar{s}}_{\bar{q},\bar{\tau}}(\Omega)} \lesssim \|f\|_{GL^{\bar{\theta}_0(\Omega)}_{\bar{p},\bar{\tau}}} \|g\|_{GL^{\bar{\theta}}_{\bar{r},\infty}[0,1]^2}$$

$$\|f * g\|_{GL^{\bar{s}}_{\bar{q},\bar{\tau}}[0,1]^2} \lesssim \|f\|_{GL^{-\bar{\theta}_0}_{\bar{p},\bar{\tau}}[0,1]^2} \|g\|_{GL^{\bar{\theta}}_{\bar{r},\infty}[0,1]^2}$$

Inverse problems of restoring the right-hHand side for the pseudoparabolic equation

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Abstract: This work is devoted to the study of inverse problems for pseudoparabolic equations. The distinctive feature of this research is that three different formulations of inverse problems described by pseudoparabolic equations are considered, where the right-hand sides depend on all independent variables, and various types of overdetermination conditions are present. Such inverse problems have not been previously studied.

We consider, in $Q_T = \{(x,t) : x \in \Omega,\ 0 < t < T\},\ \Omega \subset \mathbb{R}^n,\ n \geq 1$ the inverse problem of determining the right-hand side of a pseudoparabolic equation. The task is to determine the functions $\{u(x,t), f(t)g(x)\}$ satisfying equation

$$u_t - \Delta u_t - \Delta u - \int_0^t k(t-\tau)\Delta u(\tau)\, d\tau = f(t)g(x) \tag{1}$$

the initial condition

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{2}$$

the boundary condition

$$\left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = \left.\varphi\right|_{\partial\Omega}, \forall t \in [0,T], \tag{3}$$

the final overdetermination condition

$$u(x,T) = u_1(x), \quad x \in \Omega, \tag{4}$$

and the nonlocal overdetermination condition

$$\int_\Omega u(x,t)\, dx = e(t), \quad 0 \leq t \leq T. \tag{5}$$

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Keywords: inverse problems, pseudoparabolic equation, combyned type external influence, existence and uniqueness of solution.

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Application of the Monge-Ampere equation to geometry

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Abstract:

In the works [1] of A.D. Alexandrov, the connection between the problems of geometry "in the large" and the Monge-Ampère equation is indicated. Developing the idea of A.D. Alexandrov, Bakelman proved the existence and uniqueness of the solution to the Dirichlet problem for the elliptic Monge-Ampère equation of general type

$$z_{xx}z_{yy} - z_{xy}^2 = \varphi \tag{1}$$

in a convex domain D with the boundary condition:

$$z|_{\partial D} = f(s), \quad s \in \partial D$$

A key role in these works is played by the extrinsic curvature of a convex surface, defined as the area of its spherical image. The extrinsic curvature was first defined for convex polyhedra and obtained by limiting process for regular convex polyhedra. The entire theory of this limit process is presented in the monographs of A.D. Alexandrov and his students. The work of A. Artykbaev [3] is devoted to the proof of the existence and uniqueness of a convex surface with a given extrinsic curvature in Galilean space. The solution to this problem required defining the concept of extrinsic curvature in spaces with a degenerate metric. Therefore, in the paper a general method for constructing extrinsic curvature in spaces with projective metrics is given. The existing connection between the problem of recovering a convex surface from extrinsic curvature and the Monge-Ampère equation is also preserved for non-Euclidean spaces. Therefore, the geometric solution of the problem led to the solution of the Monge-Ampère equation in multi-connected domains with different boundary conditions. Generalizing the obtained results, we can state: **Proposition.** Problems of recovering of a convex surface by extrinsic curvature in non-Euclidean spaces are particular solutions of the Monge-Ampère equation. **Keywords:** Convex surface, extrinsic curvature, spherical mapping, projective space, Galilean space, non-Euclidean space.

2020 Mathematics Subject Classification: 35J96, 53A15, 53C45, 52A20

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Stability estimates of discrete analogues of integral geometry problems

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Abstract: The problems of integral geometry consist in finding functions defined on a certain manifold through its integrals on a certain set of submanifolds with a lower dimension. Under sufficiently general assumptions regarding the family of curves and the weight function, the problem of integral geometry is reduced to a boundary value problem for an equation of mixed type. Estimates of the stability and uniqueness of the solution of discrete analogs of this problem on the space of sufficiently smooth functions are obtained. Due to the absence of a theorem on the existence of a solution, the concept of conditional correctness of the problem is used in the work, namely, it is assumed that a solution to the differential-difference and finite-difference problems exists. The proof is carried out using the method proposed in [1-3]. The results obtained can be applied to computer tomography problems.

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Keywords: integral geometry, stability estimates, differential-difference problem, finite-difference problem

2020 Mathematics Subject Classification: 65M32, 65N21

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Stability and Hopf bifurcation analysis for a financial dynamical model with time delay

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Abstract: In this study, we consider a fractional-order financial dynamical model with time delay. We study the impact of the time delay on the stability of the model and by choosing the delay time *tau* as a bifurcation parameter, we show that Hopf bifurcation can occur as the delay time *tau* passes some critical values. Moreover, the local stability of a positive equilibrium is established . Finally, we give numerical simulations to support our theoretical results.

Keywords: Hopf bifurcation, fractional-order, time-delay, stability, finance model, numerical simulation, predictor-corrector model

2020 Mathematics Subject Classification: 34A08, 34K18, 34K20

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Riesz basis for differential-algebraic equations with boundary terms

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Abstract: Let $\mathcal{L}$ be an operator defined in $L_2(0,1)\times\mathbb{C}^s$, mapping an element $\begin{pmatrix} y(x) \\ \vec{\mu} \end{pmatrix}$ to the element $\begin{pmatrix} l(y,\vec{\mu}) \\ \vec{\theta}(y,\vec{\mu}) \end{pmatrix}$, where

$$
(1)\qquad \begin{cases} l(y,\vec{\mu}) \equiv \dfrac{d^2y(x)}{dx^2} + \varphi_1(x)y(0) + \varphi_2(x)y(1) + \displaystyle\sum_{j=1}^{s} \mu_j q_j(x) \\ + i\cdot r_1(x)\dfrac{dy(x)}{dx} + i\cdot\dfrac{dr_1(x)}{dx}y(x) + r_0(x)y(x), \\ \theta_m(y,\vec{\mu}) \equiv \displaystyle\int_0^1 y(x)G_m(x)dx + A_{1m}y(0) + A_{2m}y(1) + \sum_{j=1}^{s}\delta_{mj}\mu_j, \; m=1,\dots,s. \end{cases}
$$

The domain of the operator $\mathcal{L}$ is defined as:

$$
(2)\qquad D(\mathcal{L}) = \left\{ \begin{pmatrix} y(x) \\ \vec{\mu} \end{pmatrix} : y(x)\in W_2^2[0,1], \vec{\mu}\in\mathbb{C}^s \;\middle|\; V_j(y,\vec{\mu}) = 0, j=1,2. \right\}.
$$

where

$$
(3)\qquad \begin{cases} V_1(y,\vec{\mu}) \equiv \displaystyle\int_0^1 y(x)\theta_1(x)dx - y(0)B_{11} + y(1)B_{12} + y'(1) + \sum_{j=1}^{s}\mu_j\alpha_{1j}, \\ V_2(y,\vec{\mu}) \equiv \displaystyle\int_0^1 y(x)\theta_2(x)dx - y(0)B_{21} + y(1)B_{22} + y'(0) + \sum_{j=1}^{s}\mu_j\alpha_{2j}. \end{cases}
$$

In the first part of the talk, the form of the adjoint operator $\mathcal{L}^*$ is presented. Then, the conditions under which the operator equality $\mathcal{L}=\mathcal{L}^*$ holds are identified. For such operators, the Riesz basis property of the system of eigen-elements $\begin{pmatrix} y_n(x) \\ \vec{\mu}_n \end{pmatrix}$ in the corresponding space is established.

In the special case where $h_p(x)\equiv 0$ and $q_j(x)\equiv 0$, the adjoint operator $\mathcal{L}^*$ is studied separately. Conditions are derived under which a certain subsequence of the first components $\{y_n^*(x)\}$ of the root elements $\begin{pmatrix} y_n^*(x) \\ \vec{\mu}_n^* \end{pmatrix}$ of the operator $\mathcal{L}^*$ forms a Riesz basis in the space $L_2(0,1)$.
In this note, we primarily rely on the methods developed in our work [1].

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP23485971).

Keywords: Riesz basis, boundary conditions, eigenvalues, boundary terms, adjoint operator

2020 Mathematics Subject Classification: 34A30, 34B05, 34B10

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On the mathematical model of balanced sustainable development of the economic sector

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Ensuring sustainable development of each society requires solving the problems of ensuring a balanced, that is, a constant dynamic pace of sustainable development of economic sectors. Digitalization of this activity, in other words, its implementation with the help of advanced digital technologies, mainly begins with the development of economic and mathematical models of the corresponding problems. Observations show that the correlation dependence of the total indicator of sustainable development of the industry U with indicators of the social, environmental and economic spheres is low, that is, the modules of the correlation coefficients of these indicators are quite small. Therefore, the dependence of U of sustainable development on indicators U_1, U_2, U_3 can be considered nonlinear. In this case, the nonlinear regression equation can be sought in the following form:

$$U = A\ U_1^{\alpha 1}\ \cdot U_2^{\alpha 2} \cdot U_3^{\alpha 3} \tag{1}$$

Here A is the assessment of the total impact of other unaccounted factors that may affect the sustainable development of the industry. In the case of statistical significance of the assessment of the parameters of the regression equation (1), it can be used to predict the dynamics of sustainable development of the industry in the near future in the main areas. To do this, the elasticity coefficients (E_1, E_2, E_3) of the integral indicator U of sustainable development of the industry from the main directions of indicators U_1, U_2, U_3 are calculated:

$$E_1 = \frac{\partial U}{\partial U_1} \cdot \frac{U_1}{U} = \alpha_1,\ E_2 = \frac{\partial U}{\partial U_2} \cdot \frac{U_2}{U} = \alpha_2, E_3 = \frac{\partial U}{\partial U_3} \cdot \frac{U_3}{U} = \alpha_3, \tag{2}$$

So, in this case, the estimates $\alpha_1, \alpha_2, \alpha_3$ exactly describe the elasticity coefficients E_1, E_2, E_3. The main advantage of expressing the sustainable development of the industry in the form (1) is that the correct calculation of the values α_i $(i = 1, 2, 3)$ which reflect the contribution of the main indicators U_1, U_2, U_3 to the sustainable development of the industry, makes the integral indicator more accurate. Using the well-known Taylor formula, we obtained the following formula for assessing the relative growth of sustainable development of the industry [1]

$$\frac{\Delta U}{(U)|_{U_i=U_i^0}} \approx \left(\frac{\Delta U_1}{U_1^0} E_1 + \frac{\Delta U_2}{U_2^0} E_2 + \frac{\Delta U_3}{U_3^0} E_3 \right) \tag{3}$$

This equality shows that the vector of elasticity coefficients (E_1, E_2, E_3) of the index of economic, social and environmental components of the economic sector serves as a normative vector of relative growth in relation to its components. Also, formula (2) connects the relative growth of sustainable growth of the economic sector with the relative growth of its components. Therefore, with its help, it is possible to describe and analyze various situations of sustainable development of the economic sector. In particular, the condition of stability of relative development can be considered as the main condition of sustainable development of the industry:

$$\frac{\Delta U}{(U)|_{U_i=U_i^0}} = const > 0\ \leftrightarrow\ \left(\frac{\Delta U_1}{U_1^0} E_1 + \frac{\Delta U_2}{U_2^0} E_2 + \frac{\Delta U_3}{U_3^0} E_3 \right) = const > 0.$$

The fulfillment of condition (4) determines that the sustainable development of the economic sector depends on the relative development of its components. It also makes it possible to determine the state of sustainable development of the industry and its balance with its components. Therefore, condition (4) can be considered as a condition of stability of sustainable development of the industry.

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An approximate solution of nonlocal problem for hyperbolique equation with deviated boundary conditions

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Abstract: In this work, we introduce a model of a nonlocal partial differential equation with a deviated function in the boundary condition. The finite diffrence method with a variable space gride is applied to construct the numirical solution of our problem. The stability of the approximat solution is investigate by using Von-Neumann method.
The results of numirical expriments are presented, and are compared with analytical solution and are found to be in good argeement with each other.It is shown that the numirical solutions expected convergence to the exact one as the mesh size is reduced.

Keywords: numerical solution, Non local partial differential equation, Deviated boundary condition, Variable space grid method.

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Stability of third-order partial differential equation under nonclassical integral boundary conditions

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Abstract: This work addresses third-order partial differential equation subject to nonlocal integral and nonclassical boundary conditions. The main objective is to establish the well-posedness of the corresponding nonlocal boundary value problem. Employing an operator-theoretic framework, we derive stability theorems ensuring the continuous dependence of solutions on the input data. These general results are then applied to obtain explicit stability estimates for two specific nonlocal boundary value problems involving third-order PDEs, thereby illustrating the applicability and significance of the theoretical analysis.

Keywords: Self-adjoint operators, Positive definite operators, Stability analysis, Hilbert spaces, Nonlocal boundary value problems, Third-order partial differential equations.

2020 Mathematics Subject Classification: 35G15; 47A62

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Solvability of a boundary value problem for fractional differential equations with involution

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Abstract: Fractional calculus is an extension of classical differential and integral calculus to arbitrary (non-integer) orders of differentiation. In recent decades, interest in this field has significantly increased due to its wide range of applications, including modeling of memory effects, anomalous diffusion, and complex system dynamics.

Definition [1]. Let $[a, b]$ be a finite interval on the real axis R. If $0 < \alpha < 1$, then the left and right Caputo fractional derivatives are defined as:

$$ {}_C D_a^{\alpha} f(x) = {}_{RL} D_a^{\alpha} \left(f(x) - f(a)\right), $$

$$ {}_C D_b^{\alpha} f(x) = {}_{RL} D_b^{\alpha} \left(f(x) - f(b)\right). $$

where ${}_{RL} D_a^{\alpha} f(x)$, ${}_{RL} D_b^{\alpha} f(x)$ are the left-and right-sided Riemann–Liouville fractional derivatives of order α, $0 < \alpha < 1$.

In this work, over the interval $[0, T]$, we study a boundary value problem for an inhomogeneous fractional differential equation of order $0 < \alpha < 1$ with involution:

$$ {}_C D^{\alpha} y(x) + \varepsilon {}_C D^{\alpha} y(T - x) = f(x), \tag{1} $$

subject to the boundary condition:

$$ ay(0) + by(T) = c, \tag{2} $$

here, the function f(x) is continuous on the considered interval.

To solve the problem, the parameterization method proposed by Professor D. Dzhumabaev [2] is used. A parameter is introduced as: $\mu = y(0)$, and a substitution is made: $y(x) = u(x) + \mu$. Thus, the original boundary value problem is split into two parts: a Cauchy problem for a fractional differential equation of order $0 < \alpha < 1$, and a linear equation with respect to the introduced parameter. The parameters are chosen so that the Cauchy problem has a classical solution.

By applying appropriate transformations, a unique solution to the Cauchy problem is obtained. Based on the unique solvability of this equation with respect to the parameter, the solvability of the original boundary value problem is established.

Theorem 1. Let $\varepsilon \neq 1$. Then, the boundary value problem (1), (2) has a unique solution if and only if $q = a + b \neq 0$.

Theorem 2. Let $\varepsilon \neq 1$. If $q = a + b = 0$, then the boundary value problem (1), (2) has a unique solution if and only if the following condition holds:

$$ \int_0^T \left((T - \xi)^{\alpha - 1} - \varepsilon \xi^{\alpha - 1}\right) f(\xi)\, d\xi = \frac{c\left(1 - \varepsilon^2\right)\Gamma(\alpha)}{b}. $$

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Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

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Abstract: Problem statement

We consider an initial boundary value problem for the heat equation with a piecewise constant coefficient and with a fractional time derivative

$$Lu \equiv \left\{ \begin{array}{ll} t^{-\beta} D_t^{\alpha} u(x,t) - k_1^2 u_{xx}(x,t), & 0 < x < x_0, \\ t^{-\beta} D_t^{\alpha} u(x,t) - k_2^2 u_{xx}(x,t), & x_0 < x < l \end{array} \right\} = 0 \tag{1}$$

in the domain $\Omega = \Omega_1 \cup \Omega_2$, $\quad \Omega_1 = \{(x,t)\colon 0 < x < x_0,\ 0 < t < T\}$, $\quad \Omega_2 = \{(x,t)\colon x_0 < x < l,\ 0 < t < T\}$ (i=1,2), with initial condition

$$u(x,0) = \varphi(x), \quad 0 \le x \le l \tag{2}$$

boundary conditions of the form

$$\begin{cases} a_{11} u_x(0,t) + a_{12} u(0,t) = 0, \\ a_{21} u_x(l,t) + a_{22} u(l,t) = 0, \end{cases} \qquad 0 \le t \le T, \tag{3}$$

and with conjugation conditions

$$\begin{cases} u(x_0 - 0, t) = u(x_0 + 0, t), \\ k_1 u_x(x_0 - 0, t) = k_2 u_x(x_0 + 0, t), \end{cases} \tag{4}$$

where coefficients $k_i > 0, a_{i,j}, (i,j = 1,2)$ real numbers. $0 < \alpha < 1, |a_{11}| + |a_{12}| > 0, |a_{21}| + |a_{22}| > 0$.

By the method of separation of variables, problem (1)-(4) is reduced to a spectral problem, which is studied in detail in the work [1]. Using the results of this work, we have proven the following theorem.

Theorem. Let $\varphi(x)$ be a twice continuously differentiable function satisfying boundary conditions and conjugation conditions $a_{11}\varphi'(0) + a_{12}\varphi(0) = 0$, $\quad a_{21}\varphi'(l) + a_{22}\varphi(l) = 0$, $\varphi(x_0 - 0) = \varphi(x_0 + 0)$, $\quad k_1\varphi'(x_0 - 0) = k_2\varphi'(x_0 + 0)$. Then the function

$$u(x,t) = \sum_{n=1}^{\infty} \varphi_n X_n(x) E_{\alpha, 1+\frac{\beta}{\alpha}, \frac{\beta}{\alpha}}(-\lambda_n t^{\alpha+\beta})$$

is a unique classical solution to problem (1)-(4), where

$$\varphi_n = \int_0^l \varphi(x) Y_n(x) dx, \quad X_n(x) = C_n \begin{cases} \Phi_2(x_0, \lambda_n)\Phi_1(x, \lambda_n), 0 < x < x_0, \\ \Phi_1(x_0, \lambda_n)\Phi_2(x, \lambda_n), x_0 < x < l, \end{cases}$$

$$Y_n(x) = C_n \begin{cases} \frac{1}{k_1}\Phi_2(x_0, \lambda_n)\Phi_1(x, \lambda_n), 0 < x < x_0, \\ \frac{1}{k_2}\Phi_1(x_0, \lambda_n)\Phi_2(x, \lambda_n), x_0 < x < l, \end{cases}$$

$$\Phi_1(x, \lambda_n) = cos\Big(\frac{\sqrt{\lambda_n}}{k_1}x\Big) - \frac{a_{12}k_1}{a_{11}\sqrt{\lambda_n}} sin\Big(\frac{\sqrt{\lambda_n}}{k_1}x\Big),$$

$$\Phi_2(x, \lambda_n) = cos\Big(\frac{\sqrt{\lambda_n}}{k_2}(l - x)\Big) + \frac{a_{22}k_2}{a_{21}\sqrt{\lambda_n}} sin\Big(\frac{\sqrt{\lambda_n}}{k_2}(l - x)\Big),$$

$$C_n = \Big(\frac{1}{k_1} \Phi_2^2(x_0, \lambda_n) \int_0^{x_0} \Phi_1^2(x, \lambda_n) dx + \frac{1}{k_2} \Phi_1^2(x_0, \lambda_n) \int_{x_0}^{l} \Phi_2^2(x, \lambda_n) dx \Big)^{-\frac{1}{2}},$$

$E_{\alpha, 1+\frac{\beta}{\alpha}, \frac{\beta}{\alpha}}(z)$- generalized Mittag-Leffler function.

$$E_{\alpha, 1+\frac{\beta}{\alpha}, \frac{\beta}{\alpha}}(z) = \sum_{k=0}^{\infty} c_k z^k, c_0 = 1, c_k = \prod_{j=0}^{k-1} \frac{\Gamma(j(\alpha+\beta)+\beta+1)}{\Gamma(j(\alpha+\beta)+\alpha+\beta+1)},$$

$\Gamma(z)$-gamma function.

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On reversibility of difference relations

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Abstract: Let X be a complex Banach space, $LB(X)$ be the Banach algebra of linear bounded operators in X, and $\mathcal{F}_d = \mathcal{F}_d(\mathbb{Z}_+, X)$, $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$, be one of the Banach spaces of (one-sided) sequences of vectors from $X : l_p(\mathbb{Z}_+, X), p \in [1; \infty], c_0(\mathbb{Z}_+, X)$. Let E be a closed subspace of $X, U : \mathbb{Z}_+ \to LB(X)$ be a bounded function, and $\mathcal{D}_E$ be a closed linear relation in $\mathcal{F}_d$, i.e. $\mathcal{D}_E \in LRC(\mathcal{F}_d)$, of the form

$$\mathcal{D}_E = \big\{(x, y) \in \mathcal{F}_d \times \mathcal{F}_d : y(n) = x(n) - U(n)x(n-1), n \geqslant 1; y(0) - x(0) \in E\big\}.$$

Consider $\mathcal{D}_E$ and $\mathcal{D}_{\widetilde{E}}$ – difference relations that are constructed using the same operator function $U(n) \in LB(X), n \geqslant 1$, but different closed subspaces of initial conditions E and $\widetilde{E}$ from X, respectively. From the function $U(n), n \geqslant 1$, we construct an evolutionary family $\mathcal{U} : \Delta_{\mathbb{Z}_+} \to LB(X)$, of the form $\mathcal{U}(n, m) = U(n)U(n-1)\ldots U(m+1)$ for $n > m$ and $\mathcal{U}(n, m) = I$ for $n = m$.

If the family $\mathcal{U} : \Delta_{\mathbb{Z}_+} \to LB(X)$ admits an exponential dichotomy with splitting pairs of projection-valued functions $P, Q : \mathbb{Z}_+ \to LB(X)$ and $\widetilde{P}, \widetilde{Q} : \mathbb{Z}_+ \to LB(X)$, such that $\Im Q(0) = E$, $\Im \widetilde{Q}(0) = \widetilde{E}$, then the relations $\mathcal{D}_E$ and $\mathcal{D}_{\widetilde{E}}$ are continuously invertible (see [1]) and have the equality

$$\mathcal{D}_{\widetilde{E}} = \{(\mathcal{W}_d x, \mathcal{W}_d y) : (x, y) \in \mathcal{D}_E\} = \mathcal{W}_d^{-1}\mathcal{D}_E\mathcal{W}_d, \mathcal{D}_{\widetilde{E}}^{-1} = \mathcal{W}_d\mathcal{D}_E^{-1}\mathcal{W}_d^{-1}; \sigma(\mathcal{D}_{\widetilde{E}}) = \sigma(\mathcal{D}_E),$$

Where $(\mathcal{W}_d x)(n) = W_d(n)x(n) = (P(n) + \widetilde{Q}(n))x(n), n \in \mathbb{Z}_+$ – multiplication operator.

Keywords: difference quotient, exponential dichotomy.

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Numerical algorithm for solving the inverse problem of identifying the right-hand part of the subdiffusion differential equation with nonlocal boundary conditions

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Abstract: In this work, we consider the subdiffusion equation with nonlocal boundary conditions:

$$\begin{cases} D_t^\alpha u(x,t) - u_{xx}(x,t) = \psi(x)\eta(x,t) + f(x,t), \\ u(x,0) = 0, \quad 0 \le x \le 1, \\ u_x(0,t) - u_x(1,t) - au(1,t) = 0, \quad 0 \le t \le T, \\ u(0,t) = 0, \quad 0 \le t \le T, \end{cases}$$

where $0 < \alpha < 1$ is the order of the fractional Caputo derivative, $a > 0$.

We consider the inverse problem of finding a pair of unknown functions $[u(x,t), \psi(x)]$ using the additional final overdetermination condition $u(x,T) = \varphi(x), 0 \le x \le 1$, where $\varphi(x)$ is a known function.

For solving this inverse problem, we propose an iterative algorithm on the basis of the biconjugate gradient method and Tikhonov regularization method. For solving the auxilliary forward initial boundary value problems at each iteration of the algorithm, we apply a finite difference scheme based on the L1 and central difference approximations.

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Keywords: fractional differential equations, nonlocal problems, inverse and ill-posed problems, numerical algorithms, conjugate gradient method

2020 Mathematics Subject Classification: 35R11, 47A52, 65M30

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Stability analysis and numerical study of multipoint boundary value problems for third-oder partial differential equations

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Abstract: We investigate boundary value problems for third-order partial differential equations in a Hilbert space H involving a self-adjoint positive definite operator A. Using an operator-theoretic approach, we establish stability estimates for the corresponding solutions. In particular, we analyze three classes of boundary value problems with multipoint conditions and derive explicit stability bounds.

Furthermore, we consider a multipoint nonlocal boundary value problem for a third-order partial differential equation. To approximate its solution, we construct three-step difference schemes based on Taylor's decomposition at four points. The stability of these schemes is established, and the theoretical findings are supported by numerical experiments.

Keywords: Third-order partial differential equations, Multipoint boundary value problems, Stability analysis, Difference schemes, Numerical experiments.

2020 Mathematics Subject Classification: 35G15; 47A62

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Algorithm for numerical solution of the boundary value problem for the two-dimensional elliptic differential equation

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Abstract: In this work, we consider the two-dimensional elliptic equation in the following form:

$$\begin{cases} r(x_1)u_{x_1x_1}(x_1,x_2) + \left[p(x_1)u_{x_1}(x_1,x_2)\right]_{x_1} - q(x_1)u(x_1,x_2) = f(x_1), \\ \qquad a < x_1 < b, c < x_2 < d, \\ u(a,x_2) = f_1(x_2), c \le x_2 \le d, \\ u(b,x_2) = g_1(x_2), c \le x_2 \le d, \\ u(x_1,c) = f_2(x_1), a \le x_1 \le b, \\ u(x_1,d) = g_2(x_1), a \le x_1 \le b, \end{cases}$$

where $r(x_1) > 0$, $p(x_1) > 0$, $q(x_1) > 0$, $f_1(x_2)$, $f_2(x_1)$, $g_1(x_2)$, $g_2(x_1)$ are continuous functions, and $p(x_1)$ is differentiable.

To solve the boundary problem for this equation, we construct a finite difference scheme with a second order for both coordinates. After application of the constructed scheme, we need to solve a large SLAE with block-tridiagonal matrix. To solve this system, we use the direct method of block elimination.

This work was financially supported by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (project No. AP23487362).

Keywords: differential elliptic problem, finite difference scheme, block elimination method

2020 Mathematics Subject Classification: 35J25, 65N06

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On a time-fractional parabolic equation with Zaremba-type boundary conditions

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Abstract: This report is devoted to the time-fractional parabolic equation with Zaremba-type boundary conditions:

$$D_t^{\alpha} u(t,x) - \left(a(x)u_x(t,x)\right)_x + \sigma u(t,x) = f(t,x), \quad 0 < t < T, \quad 0 < x < l,$$

$$u_x(t,0) = 0, \quad u(t,l) = 0, \quad 0 \leq t \leq T,$$

$$u(T,x) = 0, \quad 0 \leq x \leq l.$$

Here, $D_t^{\alpha} = D_{T-}^{\alpha}$ denotes the standard Riemann–Liouville fractional derivative of order $\alpha \in (0,1)$. Also, $a(x)$ $(x \in (0,l))$ and $f(t,x)$ $(t \in (0,T),\ x \in (0,l))$ are assumed to be given smooth functions, $a(x) \geq a_0 > 0$, $\sigma > 0$.

This paper examines a time-fractional parabolic equation with Zaremba-type (mixed Dirichlet-Neumann) boundary conditions. Stable finite difference schemes are formulated, and a coercive stability estimate is established for the first-order scheme. The modified Gaussian elimination method is applied to solve both first- and second-order schemes in one-dimensional cases.

Keywords: time-fractional parabolic equations, Zaremba-type boundary conditions, Riemann-Liouville derivative, difference schemes, stability

2020 Mathematics Subject Classification: 35R11, 35K20, 35B35

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Application of artificial intelligence in university educational technologies conditions

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Abstract: Artificial intelligence (AI) in education is not a substitute for a teacher, but rather a supportive tool. It takes over routine tasks and helps create more engaging and effective learning experiences.

AI is a valuable tool that allows teachers to save time, adapt the learning process to the needs of individual students, and introduce innovative forms of learning. While some teachers spend hours creating tests and searching for materials, others are already actively using AI to accelerate lesson preparation and generate new teaching ideas.

University faculty members employ AI to analyze academic performance, update curricula, and select appropriate educational resources. The capabilities of artificial intelligence are particularly valued by teachers from the Department of Applied Mathematics and Computer Science at our university.

In the course of discussion on this topic, various approaches to the integrating AI into the educational process have been developed at universities in Turkmenistan. This indicates a growing interest and need for, the integration modern technologies into the higher education system.

Keywords: Artificial intelligence, higher education, adaptive learning, educational technologies, digital transformation, personalized learning, Turkmenistan universities

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On application of the generalized monotone iterative approach for a class of integro-differential equations of fractional type

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Abstract: We consider the following nonlinear Caputo fractional integro-differential equation with non-linear boundary conditions:

$$ {}^{C}D^{q_1}u(t) = F(t, u(t), I^{q_2}u(t)) + G(t, u(t), I^{q_3}u(t)) \quad , \tag{1} $$

$$ h(u(0), u(T)) = 0, \tag{2} $$

where $F, G \in C[J \times \mathbb{R} \times \mathbb{R}_+, \mathbb{R}]$, $u \in C^1[J, \mathbb{R}]$, $h \in C\left[\mathbb{R}^2, \mathbb{R}\right]$, $J = [0, T]$ and $0 < q_3 \leq q_2 \leq q_1 < 1$.

The monotone iterative technique, with the method of coupled upper and lower solutions, produces monotone sequences that converge uniformly and monotonically to the extremal solutions of the problem considered. In this work, we examine the existence of the solutions of the problem (1)-(2) by applying generalized monotone iterative technique to obtain minimal and maximal solutions.

Keywords: Caputo derivative, Integro-differential equation, Monotone Iterative Technique, Maximal and Minimal solutions.
2020 Mathematics Subject Classification: 34A12, 34A45, 34C11

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Boundary value problems for differential equations with nonlinear fractional variable order

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Abstract: We consider the following boundary value problem (BVP) involving fractional operator with variable order (VO)

$$(1)\quad \mathcal{D}_{0+}^{\vartheta(t,x(t))}x(t) = F(t,x(t)), \quad t \in \Omega := (0,L), 0 < L < \infty,$$
$$(2)\quad x(0) = x(L) = 0,$$

where $\mathcal{D}_{0+}^{\vartheta(t,x(t))}$ denotes Riemann-Liouville (R-L) derivative of variable order and ϑ, F are specified nonlinear functions, $1 < \vartheta_* \leq \vartheta(t,x(t)) \leq \vartheta^* < 2$.
This work introduces a boundary value problem for a nonlinear fractional differential equation characterized by the nonlinear variable order and discusses the existence and uniqueness of solutions. By employing the contraction mapping principle, we demonstrate an existence theorem in a Lebesgue space. We then employ the Schauder fixed-point theorem to establish an existence result in the weighted space of continuous functions.
Keywords: Fixed point theory, fractional differential and integral of variable order, Boundary-value problem

2020 Mathematics Subject Classification: 26A33, 47H08, 34B15, 34A08, 37C25

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Existence and uniqueness of the two-phase non-classical Stefan problem including thermoelectric effect and inner heat source in a region with varrying cross-section

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Abstract: This study presents a detailed mathematical investigation of a three-phase Stefan problem that models the simultaneous melting, solidification, and vaporization processes in electrical contact materials. The analysis incorporates a generalized heat equation that accounts for both the Thomson effect and Joule heating. The model features nonlinear thermo-physical properties and temperature - dependent coefficients across the vapor, liquid, and solid regions. Through the use of dimensionless variables and self-similar transformations, the governing equations are reduced to a boundary value problem for a system of nonlinear ordinary differential equations [1],[2]. Integral equations for the liquid and solid phases are derived, and the existence and uniqueness of their solutions are rigorously demonstrated using the Banach fixed-point theorem, assuming suitable conditions on the thermal coefficients. Sufficient criteria for contraction mappings are established, and bounds for the associated integral operators are obtained. These findings offer a solid theoretical basis for accurately simulating complex phase-change phenomena in electrical contact systems, particularly those involving localized heating and non-equilibrium thermoelectric effects.

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19675480).

Keywords: Stefan problem, Generalized heat equation, Thermoelectric effect, Joule heat source, Nonlinear thermal coefficient, Self-similar solution, Fixed point theory

2020 Mathematics Subject Classification: 80A22, 80A05, 35C11.

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Global existence result for a semilinear wave equation on quantum tori

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Abstract: Quantum tori $\mathbb{T}_\theta^d$, defined via a skew-symmetric real $d \times d$ matrix θ, are fundamental objects in noncommutative geometry (see, e.g., [1]). Also known as noncommutative tori or irrational rotation algebras, they generalize classical tori by incorporating noncommutative structures relevant in quantum mechanics and mathematical physics.

In recent years, quantum tori have also become important objects in noncommutative harmonic analysis (see, e.g., [2], [3]), where many classical tools have been successfully extended to this setting. Notably, differential operators and function spaces analogous to classical ones have been defined, enabling new approaches to classical inequalities and PDEs in the noncommutative framework.

In continuation of the ongoing developments in noncommutative harmonic analysis on quantum tori, we initiate our investigation by establishing a quantum analogue of the classical Gagliardo–Nirenberg inequality in this noncommutative setting, involving noncommutative (fractional) Sobolev spaces and L^p-spaces over the quantum tori.

As an application of this result, we study the global well-posedness of the following nonlinear damped wave equation with the fractional Laplacian of order $\frac{\nu}{2} > 0$:

$$\begin{cases} \partial_t^2 x(t) + (-\Delta_\theta)^{\frac{\nu}{2}} x(t) + b\partial_t x(t) + mx(t) = f(x(t)), & t > 0, \\ x(0) = x_0 \in H_2^{\frac{\nu}{2}}(\mathbb{T}_\theta^d), \\ \partial_t x(0) = x_1 \in L^2(\mathbb{T}_\theta^d), \end{cases}$$

where the damping term determined by $b > 0$ and with the mass $m > 0$, and the function f is assumed to satisfy certain Lipschitz type condition.

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Keywords: Quantum tori, Gagliardo–Nirenberg inequality, Nonlinear wave equations, Global well-posedness

2020 Mathematics Subject Classification: 46L52, 47L25, 35L76, 35L05

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A Competition model with nonlinear cross-diffusion

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Abstract: We study a two-species reaction–diffusion (RD) system in which the fluxes for each species involve gradients of both unknown variables. For a certain range of parameters, we demonstrate the existence of two distinct types of periodic stationary solutions. These solutions allow us to partition the eight-dimensional parameter space and identify regions—known as Turing domains—where such solutions occur. The boundaries of these regions, analogous to bifurcation points in lower dimensions, are referred to as bifurcation surfaces. It is commonly believed that these stationary solutions represent the long-time limits (as $t \to \infty$) of the solutions to the corresponding time-dependent RD system. We also provide numerical simulations that support this hypothesis, suggesting that these stationary patterns are attractors with a wide basin of attraction in the space of initial conditions.

We consider the following reaction–diffusion system:

$$\begin{cases} u_t = (uu_x + \varepsilon_1 uv_x + \varepsilon_3 vu_x)_x + u(1-u-cv) := -\dfrac{\partial}{\partial x} J_1 + u(1-u-cv), \\ v_t = (dvv_x + \varepsilon_4 uv_x + \varepsilon_2 vu_x)_x + v(a-bu-v) := -\dfrac{\partial}{\partial x} J_2 + v(a-bu-v). \end{cases} \tag{1}$$

The term "cross-diffusion" here means that at least one of the fluxes J_i contains both gradients u_x and v_x, i.e., either ε_1 or ε_2 (or both) are different from zero.

We also study stationary solutions of the form:

$$\begin{cases} (uu' + \varepsilon_1 uv' + \varepsilon_3 vu')' + u(1-u-cv) = 0, \\ (dvv' + \varepsilon_4 uv' + \varepsilon_2 vu')' + v(a-bu-v) = 0, \end{cases} \quad x \in \mathbb{R}. \tag{2}$$

One of the main difficulties in the theory of nonlinear reaction-diffusion systems containing parameters is to find the bifurcation surfaces in parameter space (a generalization of a bifurcation point in the case of one parameter), which separate domains with different solution behavior. It turns out that exact solutions can help in this respect too: the solutions we find must be physically relevant, non-negative, periodic, etc. This will be the case only if the parameters satisfy a rather complicated nonlinear algebraic system of inequalities, whose solutions define domains in parameter space.

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Keywords: Periodic stationary solutions, Pattern formation, Reaction–diffusion systems, Cross-diffusion, Stability of patterns

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Nonlinear inverse problems for parabolic equations

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Abstract: Let us consider the following nonlinearly degenerate domain

$$\Omega = \{x, t \mid \varphi_1(t) < x < \varphi_2(t),\ 0 < t < T < \infty\},$$

with the cross-section $\Omega_t = \{\varphi_1(t) < x < \varphi_2(t)\}$ for a fixed value of the time variable $t \in (0, T)$, and for which the condition

$$\varphi_1(0) = \varphi_2(0),$$

is satisfied.

In the domain Ω, we study the following inverse problems for the Burgers' equation:

$$\partial_t u(x,t) + u(x,t)\partial_x u(x,t) - \nu\partial_x^2 u(x,t) = \lambda(t)u(x,t) + w(t)f(x,t),$$

with various combinations of unknowns and boundary conditions.

The main purpose of this work is to determine additional conditions under which these inverse problems are uniquely solvable.

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Keywords: Burgers' equation, inverse problem, degenerate domain, Galerkin method

2020 Mathematics Subject Classification: 35K55, 35R30, 35R37

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Weighted estimates for some class of quasilinear operators

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Abstract: Let $0 < q, p, r < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$. Let $u = \{u_i\}_{i=1}^{\infty}$ and $v = \{v_i\}_{i=1}^{\infty}$ be weight sequences, i.e., positive sequences of real numbers. We denote by $l_{p,v}$ the space of sequences $f = \{f_i\}_{i=1}^{\infty}$ of non-negative real numbers such that $\|vf\|_p = \left(\sum_{i=1}^{\infty} |v_i f_i|^p\right)^{\frac{1}{p}} < \infty$.

For any non-negative $f \in l_{p,v}$ we consider the following iterated discrete Hardy-type inequality with three weights

$$\left(\sum_{n=1}^{\infty} u_n^q (K^{\pm} f)_n^q\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty} (v_i f_i)^p\right)^{\frac{1}{p}}, \tag{1}$$

where C is a positive constant independent of f and K is a quasilinear operators defined as follows

$$(K^- f)_n = \left(\sum_{k=n}^{\infty} a_{k,n} \left(\sum_{i=k}^{\infty} f_i\right)^r\right)^{\frac{1}{r}}, \quad (K^+ f)_n = \left(\sum_{k=1}^{n} a_{n,k} \left(\sum_{i=1}^{k} f_i\right)^r\right)^{\frac{1}{r}}.$$

where $a_{k,n}$ is a non-negative matrix. This type of operator was studied for the first time in [1].

We provide a characterization of the inequality (1) in the general case of the matrix when $0 < p \leq r < q < \infty$. For the parameter ratio $0 < r < q < p < \infty$, we have been studied the inequality (1) under the following matrix assumption: there exist $d \geq 1$, a sequence of positive numbers $\{\omega_k\}_{k=1}^{\infty}$ and a non-negative matrix $(b_{i,j})$, where $b_{i,j}$ is almost non-decreasing in i and almost non-increasing in j such that the inequalities

$$\frac{1}{d}(a_{i,k} + b_{k,j}\omega_i) \leq a_{i,j} \leq d(a_{i,k} + b_{k,j}\omega_i)$$

hold for all $i \geq k \geq j \geq 1$.

Furthermore, we explore the significant impacts of our results on the analysis of bilinear inequalities, demonstrating their practical applications and importance in the field.

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Keywords: Hardy-type operator, quasilinear operator, weight sequence, weighted Lebesgue space

2020 Mathematics Subject Classification: 26D15, 26D20

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A hybrid stochastic–machine learning approach to financial prediction

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Abstract: We propose a hybrid stochastic–machine learning framework for financial time series prediction, applied specifically to modeling and forecasting gold prices. The asset price process S_t is modeled using a stochastic differential equation (SDE) driven by both a Brownian motion and a continuous-time Markov chain (CTMC), which captures the regime-switching behavior observed in financial markets. The price process satisfies in general the SDE:

$$dS_t = g(\mu(Z_t))S_t\,dt + f(\sigma(Z_t))S_t\,dB_t, \quad S_0 = x > 0,$$

where Z_t is a Markov jump process with a finite state space, and the functions $g(\mu(Z_t))$ and $f(\sigma(Z_t))$ represent the regime-dependent drift and volatility, respectively. As an application, We consider a mean-reverting formulation for the log-price:

$$d(\log S_t) = \alpha(\mu(Z_t) - \log S_t)\,dt + \sigma(Z_t)\,dB_t,$$

with α representing the speed of mean reversion and $\mu(Z_t)$ the regime-dependent long-term mean. To estimate the model parameters, we employ a neural network-based machine learning approach. The parameters to be estimated include:

$$\mu_i \text{ and } \sigma_i, \; i = 1, 2, \ldots, N$$

as well as the transition rate matrix Q of the CTMC governing Z_t. Using historical gold price data, we calibrate the model and use the estimated parameters for future price prediction. Comparisons with real data demonstrate the improved performance and predictive accuracy of our hybrid model over traditional approaches. This work illustrates the effectiveness of integrating stochastic modeling with machine learning for financial forecasting tasks.

Keywords: Stochastic processes, gold prices, neural network, CTMC.

2020 Mathematics Subject Classification: 60H10, 68T07, 91G20, 91G80

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On the solvability of boundary value problems with general conditions for the triharmonic equation in a ball

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Abstract: One of the effective methods for representing solutions to boundary value problems for elliptic equations is a method based on constructing the Green's function of the problem. Many works are devoted to constructing the Green's function in explicit form for various classical boundary value problems.

The following scientific results were obtained in this report:
conditions for the solvability of boundary value problems with general conditions for the triharmonic equation in a unit multidimensional ball $S = \{x \in \mathbb{R}^n : |x| < 1\}$ were found in terms of a polynomial that does not have integer roots; a theorem on the existence of a solution to the original problem with minimal requirements for the smoothness of the boundary data was proved; an integral representation of the solution to this problem without the Green's function through solutions of three harmonic functions was given [1]. As an analogue of the Almansi formula.

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Keywords: Green's function, triharmonic equation, Dirichlet-2 problem, boundary value problem with general conditions, integral representation of the solution

2020 Mathematics Subject Classification: 31B30, 35J08, 35J40

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The Goursat - Darboux system with nonlocal boundary conditions

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Abstract: We consider nonlocal boundary conditions for the Goursat â“ Darboux system in the domain $Q = [0,T] \times [0,l]$

$$z_{tx} = f\left(t, x, z\left(t, x\right)\right), \ (t, x) \in Q, \tag{1}$$

$$Az\left(0, x\right) + \int_0^T n\left(t\right) z\left(t, x\right) dt = \varphi\left(x\right), \ \ x \in [0, l], \tag{2}$$

$$Bz\left(t, 0\right) + \int_0^l m\left(x\right) z\left(t, x\right) dt = \psi\left(t\right), \ \ t \in [0, T]. \tag{3}$$

Here $z \in R^n$, $f : Q \times R^n \to R^n$ is continuous on $Q \times R^n$, $\det\left(A + \int_0^T n\left(t\right) dt\right) \neq 0, \det\left(B + \int_0^l m\left(x\right) dx\right) \neq 0$ and $b\varphi\left(0\right) + \int_0^l m\left(x\right)\varphi\left(x\right) dx = A\psi\left(0\right) + \int_0^T n\left(t\right)\psi\left(t\right) dt$

Theorem. A problem (1)-(3) is equivalent to the following integral equation:

$$z\left(t, x\right) = \left[B + \int_0^l m\left(x\right) dx\right]^{-1} \psi\left(t\right) + \left[A + \int_0^T n\left(t\right) dt\right]^{-1} \varphi\left(x\right) -$$

$$- \left[B + \int_0^l m\left(x\right) dx\right]^{-1} \left[A + \int_0^T n\left(t\right) dt\right]^{-1} \left[B\varphi\left(0\right) + \int_0^l m\left(x\right)\varphi\left(x\right) dx\right] +$$

$$+ \int_0^T \int_0^l G\left(t, x, \tau, s\right) f\left(\tau, s, z\right) d\tau\, ds.$$

where

$$G\left(t, x, \tau, s\right) = \left[B + \int_0^l m\left(x\right) dx\right]^{-1} \left[A + \int_0^T n\left(t\right) dt\right]^{-1} \times$$

$$\times \begin{cases} \left[A + \int_0^\tau n\left(\alpha\right) d\alpha\right] \left[B + \int_0^s m\left(\beta\right) d\beta\right], & 0 \le \tau \le t,\ 0 \le s \le x, \\ - \left[A + \int_0^\tau n\left(\alpha\right) d\alpha\right] \int_s^l m\left(\beta\right) d\beta, & 0 \le \tau \le t,\ x < s \le l, \\ - \left[B + \int_0^s m\left(\beta\right) d\beta\right] \int_\tau^T n\left(\alpha\right) d\alpha, & t < \tau \le T, 0 \le s \le x, \\ \int_\tau^T n\left(\alpha\right) d\alpha \int_s^l m\left(\beta\right) d\beta, & t < \tau \le T, x < s \le l. \end{cases}$$

Similar problems have been considered in [1],[2].

Keywords: Nonlocal problem, integral and point boundary conditions, Goursat - Darboux system

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Quasi-Synchronization of Fractional-Order Bidirectional Associative Memory Neural Networks with Time-Varying Delays

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Abstract: This work addresses the synchronization phenomenon in fractional-order bidirectional associative memory (BAM) neural networks with time-varying delays. We first construct a master"slave model that includes external disturbances and memory-based feedback control. The synchronization errors are formulated through fractional differential dynamics, capturing the influence of delay and feedback memory. By developing a suitable Lyapunov functional and applying fractional-order stability lemmas involving the Mittag-Leffler function, we derive linear matrix inequality conditions that ensure synchronization to a bounded region.

The proposed results highlight the influence of fractional dynamics on the convergence behavior and synchronization performance of BAM neural networks.

Numerical simulations confirm that the designed memory-based controllers achieve reliable quasi-synchronization under delays dependent on time and external forcing.

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR21882172).

Keywords: Fractional-order neural networks, finite-time synchronization, feedback control, Lyapunov function approach, Mittag-Leffler function

2020 Mathematics Subject Classification: 92B20, 93D05, 33E12, 93C15

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